1. Some experimental results

(a) The electric charge can be negative, zero, or positive.

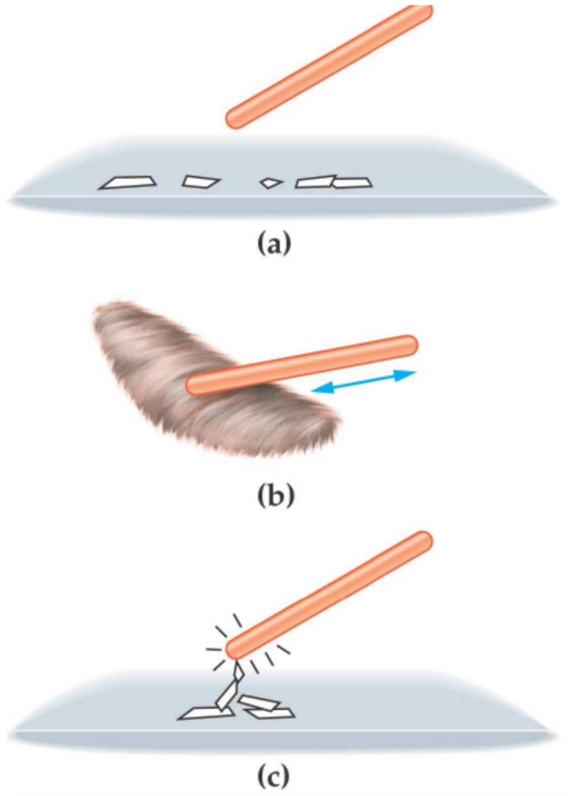
Empirically it was known since ancient times that if amber is rubbed on fur, it acquires the property of attracting light objects such as feathers. This phenomenon was attributed to a new property of matter called “electric charge”. (electron is the Greek name for amber) More experiments show that they are two distinct type of electric charge: positive (color code: red), and negative (color code: black). The names “positive” and “negative” were given by Benjamin Franklin.

((**Note**)) **Amber: Wikipedia** ηλεκτρον (Electron)

The Greek name for amber was ηλεκτρον (Electron) and was connected to the Sun God, one of whose titles was Elector or the Awakener. It is discussed by Theophrastus, possibly the first ever mention of the material, and in the 4th century BC. The modern term electron was coined in 1891 by the Irish physicist George Stoney, using the Greek word for amber (and which was then translated as electrum) because of its electrostatic properties and whilst analyzing elementary charge for the first time. The ending -on, common for all subatomic particles, was used in analogy to the word ion.

((**Note**)) **Amber**

Amber is fossil tree resin, which is appreciated for its color and beauty. Good quality amber is used for the manufacture of ornamental objects and jewelry. Although not mineralized, it is often classified as a gemstone. A common misconception is that amber is made of tree sap; it is not. Sap is the fluid that circulates through a plant's vascular system, while resin is the semi-solid amorphous organic substance secreted in pockets and canals through epithelial cells of the plant. Because it used to be soft and sticky tree resin, amber can sometimes contain insects and even small vertebrates. Semi-fossilized resin or sub-fossil amber is known as copal.



(a) Uncharged amber rod exerts no force on papers
(b) Amber rod is rubbed against a dry cloth (a fur)
(c) Amber rod becomes charged and attracts the papers.

(1) The electric charge on a glass rod rubbed with silk is positive.
(2) The electric charge on an amber (plastic) rod rubbed with fur is negative.

((**Note**)) Rubber rubbed with cat fur: rubber becomes negative, while the fur becomes positive.

Amber rod	(-)
Plastic rod	(-)
Rubber	(-)
Glass rod	(+)

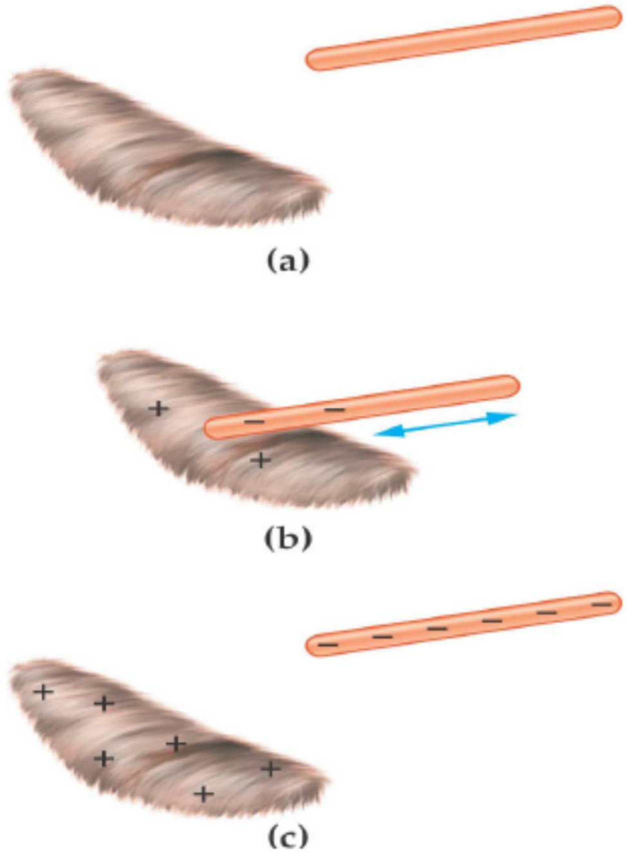
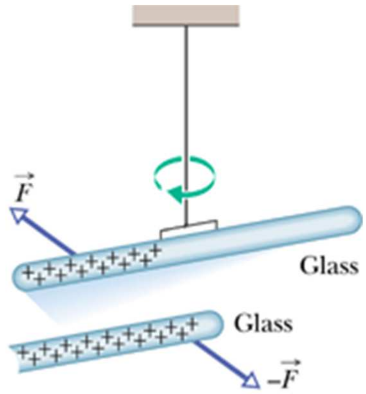


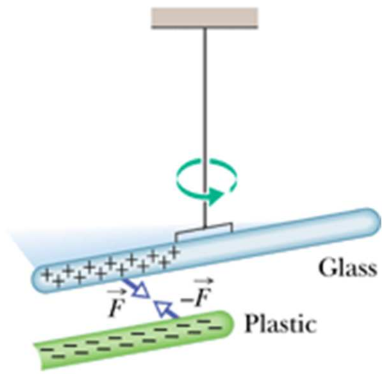
Fig. Plastic rod rubbed with fur

(b) Further experiments on charged objects showed that:

1. Charges of the same type (either both positive or both negative) repel each other.
2. Charges of opposite type on the other hand attract each other.
3. The force direction allows us to determine the sign of an unknown electric charge



(a)



(b)

2. Charge is quantized

An important experiment in which the charge of small oil droplets was determined was carried out by Millikan. Millikan discovered that the charge on the oil droplets was always a multiple of the charge of the electron (e, the fundamental charge). For example, he observed droplets with a charge equal to +/- e, +/- 2 e, +/- 3 e, etc., but never droplets with a charge equal to +/- 1.45 e, +/- 2.28 e, etc. The experiments strongly suggested that the electric charge, q, is said to be quantized. q is the standard symbol used for charge as a variable. Electric charge exists as discrete packets

$$q = ne$$

where n is an integer and e is the fundamental unit of charge.

$$e = 1.602176487 \times 10^{-19} \text{ C}$$

For electron	$q = -e$
For proton	$q = +e$
For neutron	$q = 0$

The SI Unit of charge is the coulomb. How many electrons are there to form 1 C? The answer is

$$\frac{1C}{e} = \frac{1}{1.602176487\times10^{-19}} = 6.24\times10^{18}$$

1 μC = 10^{-6} C	(μ: micro)
1 nC = 10^{-9} C	(n: nano)
1 pC = 10^{-12} C	(p: pico)
1 fC = 10^{-15} C	(f: femto)
1 aC = 10^{-18} C	(a: atto)

((**Note**))
Relation between 1 C (SI units) and 1 esu (cgs gaussian unit of charge, electrostatic unit)

We consider a force between two charges with q = 1 C. The separation between two charges is r = 1 m.

$$F_{SI} = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{(1C)^2}{4\pi\varepsilon_0 (1m)^2} \qquad [\text{N}].$$

In cgs units, the corresponding force between A (esu) [=1 C] is

$$F_{cgs} = \frac{q^2}{r^2} = \frac{(A\ \text{esu})^2}{(100cm)^2} \qquad [\text{dyne}]$$

Note that $F_{SI} = F_{cgs}$ and 1N = 10^5 dyne. Then we have

$$\frac{1}{4\pi\varepsilon_0}\times10^5 = \frac{A^2}{10^4}, \qquad \text{or} \qquad A = \sqrt{\frac{1}{4\pi\varepsilon_0}\times10^9} = 2.99792\times10^9$$

So we have

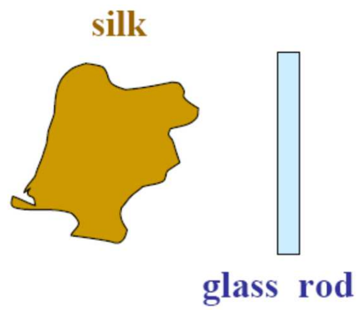
1C = 2.99792 x 10^9 esu

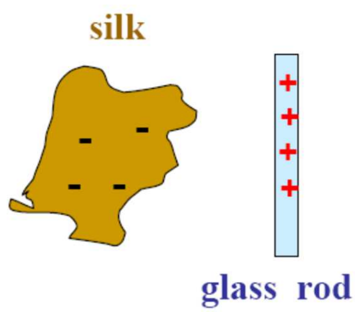
The charge of electron is

q_e = 1.60217664 x 10^{-19} C= 4.80320425 x 10^{-10} esu.

3. Charge is conserved

(a)



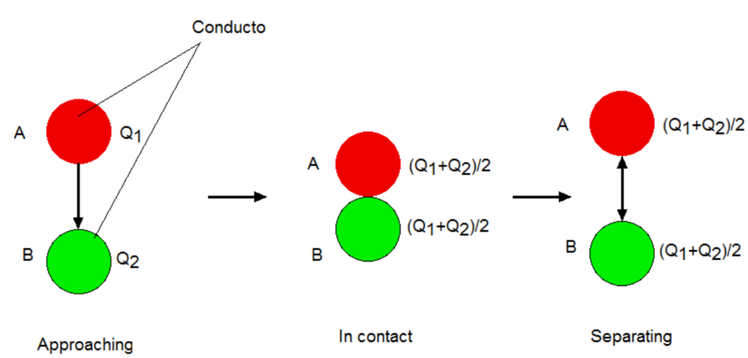


Consider a glass rod and a piece of silk cloth (both uncharged) shown in the upper figure. If we rub the glass rod with the silk cloth we know that positive charge appears on the rod (see the figure). At the same time an equal amount of negative charge appears on the silk cloth, so that the net rod-cloth charge is actually zero. This suggests that rubbing does not create charge *but* ***only transfers it from one body to the other***, thus upsetting the electrical neutrality of each body. Charge conservation can be summarized as follows: In any process the charge at the beginning equals the charge at the end of the process.

The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never change.

((**Example-2**))

We consider two identical sphere conductors which are actually well separated from one another. (Hint of HW-12)). The sphere A (with an initial charge of Q_1) is touched to sphere B (with an initial charge of Q_2) and then they are separated.



(b) Some concepts

Due to the movement of electrons, charge is transferred from one object to another.

Positive ion: the atom that loses an electron is said to be a positive ion;
Negative ion: the atom that receives an extra electron is said to be a negative ion.

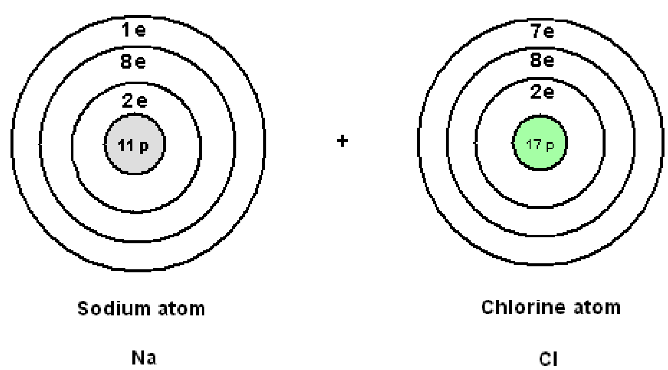
H	$(1s)$
He	$(1s)^2$
Li	$(1s)^2\vert(2s)^1$
Ba	$(1s)^2\vert(2s)^2$
B	$(1s)^2\vert(2s)^2(2p)^1$
C	$(1s)^2\vert(2s)^2(2p)^2$
N	$(1s)^2\vert(2s)^2(2p)^3$
O	$(1s)^2\vert(2s)^2(2p)^4$
F	$(1s)^2\vert(2s)^2(2p)^5$
Ne	$(1s)^2\vert(2s)^2(2p)^6\vert$
Na	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^1$
Mg	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2$
Al	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^1$
Si	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^2$
P	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^3$
S	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^4$
Cl	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^5$
Ar	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^6$
K	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^6(3d)^1$
Ca	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^6(3d)^2$

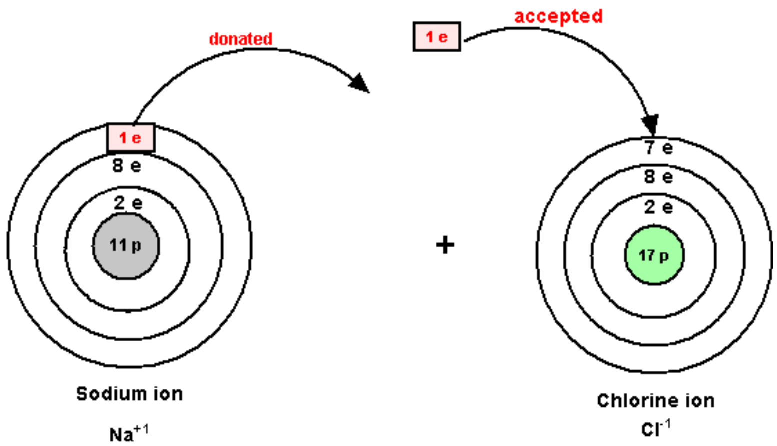
Na^+ (sodium ion)

Na	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^1$	(11 electrons)
Na^+	$(1s)^2\vert(2s)^2(2p)^6\vert$	(10 electrons)

Cl^- (chloride ion)

Cl	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^5$	(17 electrons)
Cl^-	$(1s)^2\vert(2s)^2(2p)^6\vert(3s)^2(3p)^6$	(18 electrons)





4. Coulomb's law

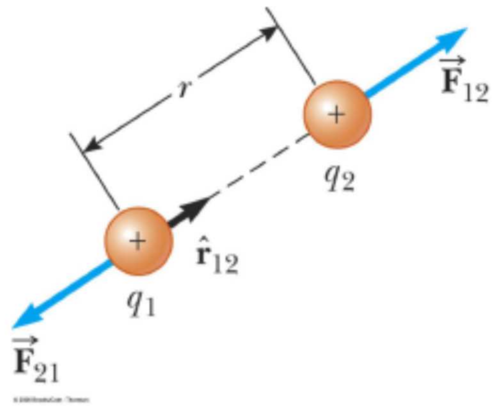
Charles-Augustin de Coulomb (June 14, 1736, Angoulême, France – August 23, 1806, Paris, France)

He was a French physicist. He is best known for developing Coulomb's law: the definition of the electrostatic force of attraction and repulsion. The SI unit of charge, the coulomb, was named after him.

The interaction between electric charges at rest is described by Coulomb's law. Two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square if the distance between them.

We can state this compactly in vector form

$$\boldsymbol{F}_{12} = k_e \frac{q_1 q_2}{r^2} \boldsymbol{e}_{12}$$



Here q_1 and q_2 are numbers (scalars) giving the magnitude and sign of the respective charges, $\boldsymbol{e}_{12}$ is the unit vector in the direction from charge 1 to charge 2, and $\boldsymbol{F}_{12}$ is the force acting on charge 2. Note that

$$\boldsymbol{F}_{21} = -\boldsymbol{F}_{12} .$$

The constant of proportionality (k_e) is written as

$$k_e = \frac{1}{4\pi\varepsilon_0} = c^2 \times 10^{-7} = 8.98755 \text{ x } 10^9 \text{ N m}^2/\text{C}^2 \text{ (or V m/C)}$$

where c is the speed of light,

$$c = 2.99792458 \text{ x } 10^8 \text{ m/s}$$

Note that ε_0 is the permittivity of free space and μ_0 is the permeability of free space,

$$\varepsilon_0 = 8.8541878176 \times 10^{-12} \frac{C^2}{Nm^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ (N/A}^2\text{)}$$

The coulomb is an extremely large unit. The force between two charges of 1 C each a distance of 1 m apart is

$$F = \frac{1}{4\pi\varepsilon_0} \frac{1C \times 1C}{1m^2} = 8.98755 \times 10^9 N$$

((**Note**)) It is easy for you to memorize the value of k_e.

$$k_e = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \text{ (or V m/C)}$$

The quantity ε_0 is called the permittivity constant.

$$\varepsilon_0 = 8.854187817\text{x}10^{-12}\ \text{C}^2/(\text{N m}^2).$$

((Note))

$$\frac{Nm^2}{C^2} = \frac{Nm}{C}\frac{m}{C} = \frac{J}{C}\frac{m}{C} = \frac{VAs}{As}\frac{m}{C} = \frac{Vm}{C}$$

$\text{Nm} = \text{J}$, $\quad C = A\ s$

$W = VA$ $\quad J = W\ s = VAs$

where

J (Joule), A (Ampere), V (Volt), C (Coulomb),
s (second), N (Neuton), and W (Watt).

((**Note**))

The SI unit of charge is coulomb. The coulomb unit is derived from the SI unit A (Ampere) for the electric current i. The current i is the rate dq/dt at which the amount of charge (dq) moves past a point or through a region in time dt (second).

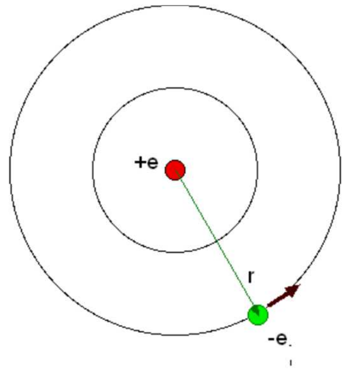
$$i = \frac{dq}{dt}.$$

This relation implies that.

$$1\text{C} = (1\text{A})(1\text{s})$$

5. Bohr model

We now consider the Bohr model shown in this figure. The system consists of a proton and an electron. These two particles are coupled with an attractive Coulomb interaction.



The electrical force between the electron (charge q_1 = -e) and proton (charge q_2 = e) is found from Coulomb's law,

$$F_e = \frac{k_e q_1 q_2}{r_B^{\ 2}} = 8.19\text{x}10^{-8}\ \text{N}$$

where e = 1.602176487 x 10^{-19} C and r_B is the Bohr radius given by

r_B = 5.2917720859 x 10^{-11} (m) =0.52917720859 Å.

This can be compared with the gravitational force between the electron and proton

$$F_g = \frac{G m_e m_p}{r_B^{\ 2}} = 3.63153\text{x}10^{-47}\ \text{N}$$

What is the angular frequency ω for electrons rotating the circular orbit?

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_B^{\ 2}} = m\frac{v^2}{r_B} = m r_B \omega^2$$

$$\omega = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m r_B^{\ 3}}} = 4.13414 \times 10^{16} \text{ rad/s}$$

where m is the mass of electron, m = 9.1093821545 x 10^{-31} kg.

The period is

$$T = \frac{2\pi}{\omega} = 1.51983 \times 10^{-16} s$$

((**Note**))

An important difference between the electric force and the gravitational force is that the gravitational force is **always *attractive***, while the electric force can **be *repulsive*, or attractive, depending on the charges of the particles**.

((**Mathematica**))

```
Clear["Global`*"];
rule1 = {g → 9.80665, G → 6.6742867 10^-11,
   me → 9.1093821545*10^-31, qe → 1.602176487*10^-19,
   rB → 0.52917720859*10^-10,   c → 2.99792458*10^8,
   μ0 → 12.566370614*10^-7, ϵ0 → 8.854187817*10^-12,
   mp → 1.672621637*10^-27};
```

$$k = \frac{1}{4\,\pi\,\epsilon 0} \;/.\; \text{rule1}$$

8.98755×10^{9}

$$Fe = \frac{1}{4\,\pi\,\epsilon 0}\,\frac{qe^2}{rB^2} \;//.\; \text{rule1}$$

8.23872×10^{-8}

$$Fg = G\,\frac{mp\;me}{rB^2} \;//.\; \text{rule1}$$

3.63153×10^{-47}

```
Fe / Fg // Simplify
```

2.26867×10^{39}

$$\omega 1 = \sqrt{\frac{qe^2}{4\,\pi\,\epsilon 0\;me\;rB^3}} \;/.\; \text{rule1}$$

4.13414×10^{16}

$$T1 = \frac{2\,\pi}{\omega 1}$$

1.51983×10^{-16}

6. Conductors and insulators

(a) Conductors

A conductor is a material that permits the motion of electric charge through its volume. Examples of conductors are copper, aluminum and iron. ***An electric charge placed on the end of a conductor will spread out over the entire conductor until an equilibrium distribution is established.***

(b) Insulators

In contrast, electric charge placed on an insulator stays in place: an insulator (like ***glass, rubber and mylar***) does not permit the motion of electric charge.

(c) Superconductors

Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance. In these chapters we discuss only conductors and insulators.

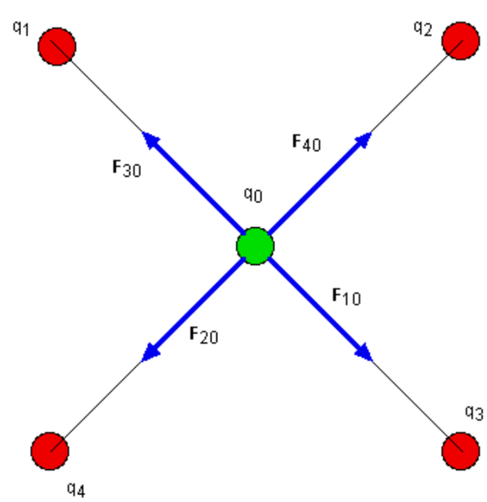
7. Principle superposition

When there are more than two charges present – the only really interesting times-we must supplement the Coulomb's law with one other fact of nature: the force on any charge is the vector sum of the Coulomb forces from each of the other charges. This fact is called "the principle of superposition." That is all there is to electrostatics. If we combine the Coulomb's law and the principle of superposition, there is nothing else.

Suppose we have some arrangement of charges q_1, q_2, q_3, …, q_N, fixed in space. From the principle of superposition, the resultant force on the charge q_0 is expressed by

$$\boldsymbol{F}_0 = \sum_{j=1}^{N} \boldsymbol{F}_{j0} = \sum_{j=1}^{N} \frac{1}{4\pi\varepsilon_0} \frac{q_0 q_j}{r_{j0}^{\;2}} \boldsymbol{e}_{j0}$$

((**Example**))



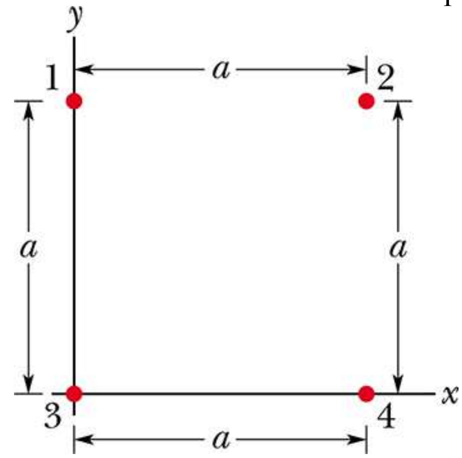
The resultant force $\boldsymbol{F}_0$ on the charge q_0 is given by

$$\boldsymbol{F}_0 = \boldsymbol{F}_{10} + \boldsymbol{F}_{20} + \boldsymbol{F}_{30} + \boldsymbol{F}_{40}$$

8. Typical example

8.1 Problem 21-8 (SP-21)

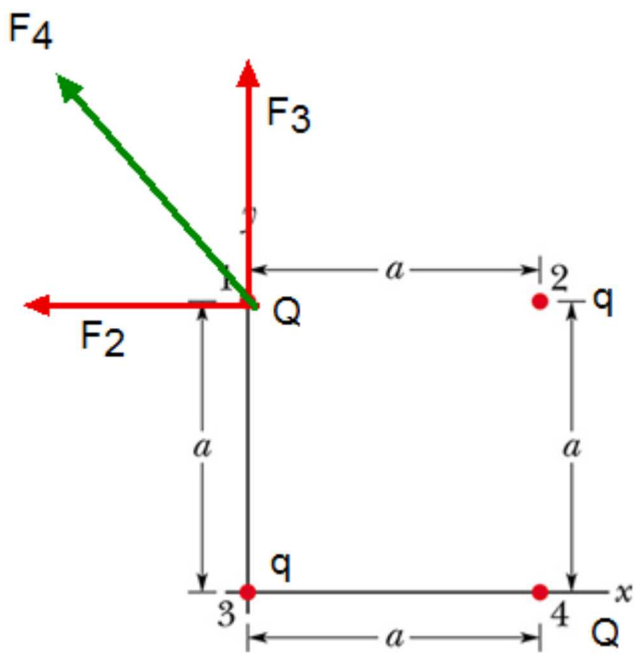
In Fig., four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. (a) What is Q/q if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of q that makes the net electrostatic force on each of the four particles zero? Explain.



((**Solution**))

$q_1 = q_4 = Q$

$q_2 = q_3 = q$



$$F_3 = F_2 = \frac{Qq}{4\pi\varepsilon_0 a^2}$$
$$F_4 = \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)^2} = \frac{Q^2}{8\pi\varepsilon_0 a^2}$$

We find that $\boldsymbol{F}_3 + \boldsymbol{F}_2$ has only the diagonal component from the symmetry.

$$(\boldsymbol{F}_3 + \boldsymbol{F}_2)_{diagonal} = \frac{Qq}{4\pi\varepsilon_0 a^2}(2\cos 45°) = \frac{\sqrt{2}Qq}{4\pi\varepsilon_0 a^2}$$
$$(\boldsymbol{F}_4)_{diagonal} = \frac{Q^2}{8\pi\varepsilon_0 a^2}$$

If the net electrostatic force on particle is zero, we have

$$\frac{\sqrt{2}Qq}{4\pi\varepsilon_0 a^2} + \frac{Q^2}{8\pi\varepsilon_0 a^2} = 0$$

or

$$2\sqrt{2}Qq + Q^2 = 0$$
$$Q(2\sqrt{2}q + Q) = 0$$

(a) $\dfrac{Q}{q} = -2\sqrt{2}$

(b) The net electrostatic force on the charge q_2 is

$$(\boldsymbol{F}_1 + \boldsymbol{F}_4)_{diagonal} = \frac{Qq}{4\pi\varepsilon_0 a^2}(2\cos 45°) = \frac{\sqrt{2}Qq}{4\pi\varepsilon_0 a^2}$$
$$(\boldsymbol{F}_3)_{diagonal} = \frac{q^2}{4\pi\varepsilon_0 a^2}$$

The net electrostatic force on the change q_2 is equal to zero,

$$\frac{\sqrt{2}Qq}{4\pi\varepsilon_0 a^2} + \frac{q^2}{4\pi\varepsilon_0 a^2} = 0$$
$$q(\sqrt{2}Q + q) = 0$$

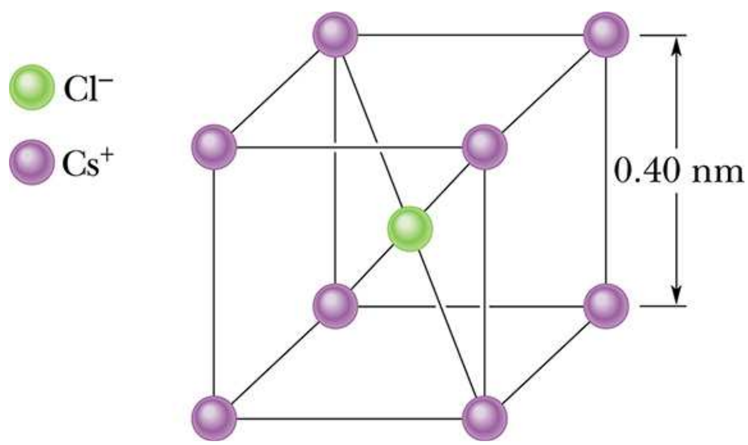
or

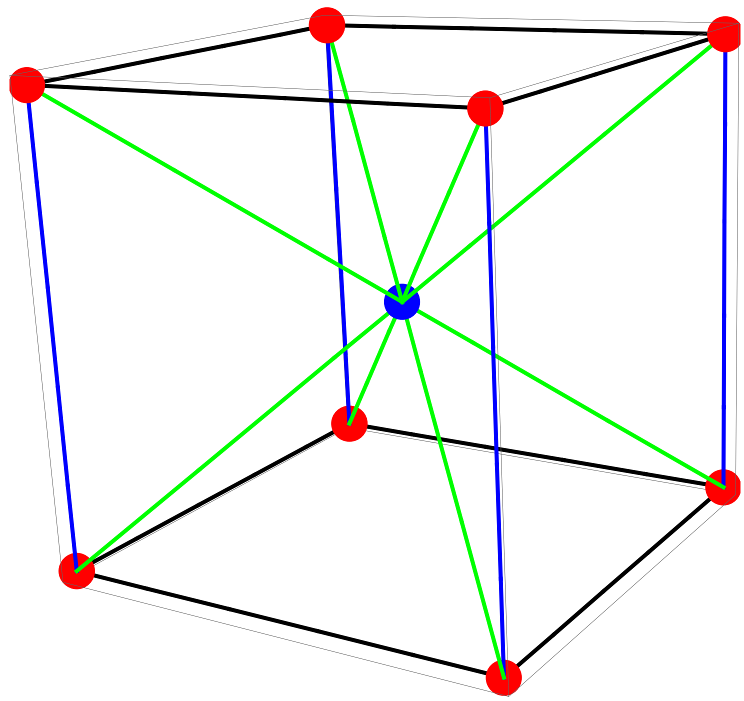
$$\frac{Q}{q} = -\frac{1}{\sqrt{2}}$$

This ratio Q/q is inconsistent with that obtained previously. So it is impossible to have such a given situation.

8.2 **Problem 21-35 (S-21)**

In crystals of the salt cesium chloride (CsCl), cesium ions Cs^+ form the eight corners of a cube and a chlorine ion Cl^- is at the cubes (Fig.). The edge length of the cube is 0.40 nm. The Cs^+ ions are each deficient one electron (and thus each has a charge of -*e*), and the Cl^- ion has one excess electron (and thus has a charge of –*e*). (a) What is the magnitude of the net electrostatic force exerted on the Cl^- ion by the eight Cs+ ions at the corners of the cube? (b) If one of the Cs+ ions is missing, the crystal is said to have a defect; what is the magnitude of the net electrostatic force exerted on the Cl^- ion by the seven remaining Cs^+ ions?





((**WileyPlus**))

35. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = \left(\sqrt{3}/2\right)a$. The force has magnitude

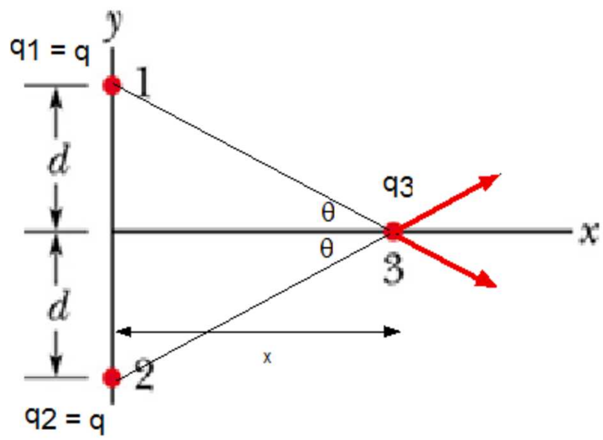
$$F = k\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{(3/4)\left(0.40\times10^{-9}\,\text{m}\right)^2} = 1.9\times10^{-9}\,\text{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

9. Hint of HW-21

9.1 Problem 21-21 (Hint)***

In Fig., particles 1 and 2 of chare $q_1 = q_2 = +3.20 \times 10^{-19}$ C are on a y axis at distance $d = 17.0$ cm from the origin. Particle 3 of charges $q_3 = +6.40 \times 10^{-19}$ C is moved gradually along the x axis from $x = 0$ to $x = +5.0$ m. At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?



((**Solution**))

$q_1 = q_2 = q = 3.20 \times 10^{-19}$ C
$q_3 = 6.40 \times 10^{-19}$ C
$d = 17$ cm
$0 \le x \le 5.0$ m

From the symmetry, $F_y = 0$.

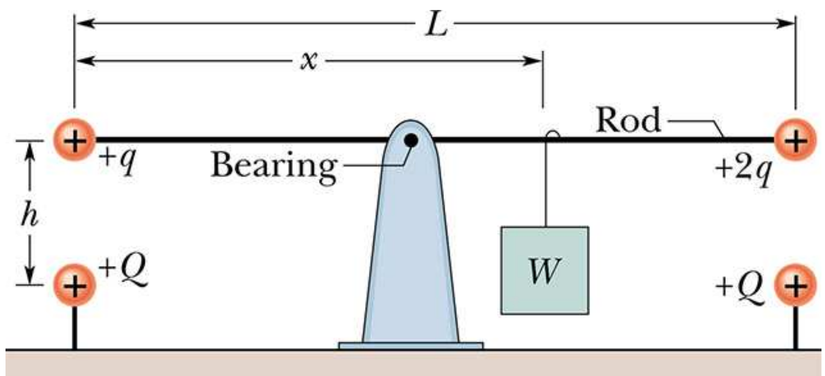
$$F_x = \frac{2qq_3}{4\pi\varepsilon_0}\frac{1}{(x^2+d^2)}\frac{x}{\sqrt{x^2+d^2}} = \frac{2qq_3}{4\pi\varepsilon_0}\frac{x}{(x^2+d^2)^{3/2}}$$

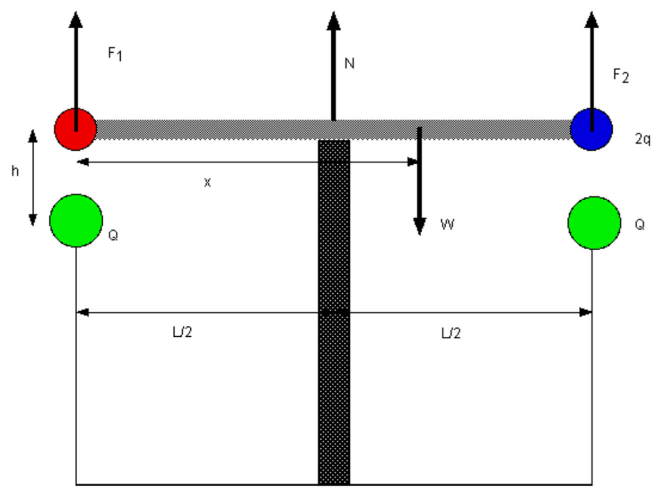
$$= \frac{2qq_3}{4\pi\varepsilon_0 d^2}\frac{\frac{x}{d}}{(\frac{x^2}{d^2}+1)^{3/2}} = \frac{2qq_3}{4\pi\varepsilon_0 d^2}\frac{t}{(t^2+1)^{3/2}}$$

where $t = x/d$.

9.2 Problem 21-44 (Hint)

Figure shows a long, nonconducting, massless rod of length L, pivoted at its center and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q. (a) Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?



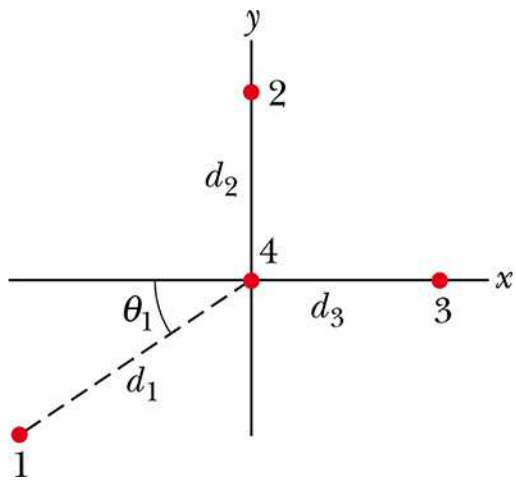


Free-body diagram

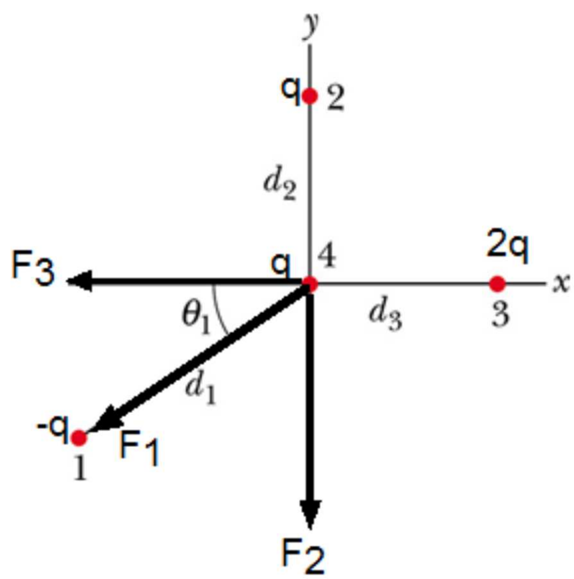
We set up the equations from the conditions, $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau = 0$ around the origin.

9.3 Problem 21-60 (HW-21, Hint) SSM

In Fig., what are the (a) magnitude and (b) direction of the net electrostatic force on particle 4 due to the other three particles? All four particles are fixed in the *xy* plane, and q_1 = -3.20 x 10^{-19} C, q_2 = +3.20 x 10^{-19} C, q_3 = + 6.40 x 10^{-19} C, q_4 = +3.20 x 10^{-19} C, q_1 = 35.0°, d_1 = 3.00 cm, and $d_2 = d_3$ = 2.00 cm.



((**Solution**))
$q_1 = -3.20 \times 10^{-19}$ C = $-q$
$q_2 = q$
$q_3 = 2\,q$
$q_4 = q$
$\theta_1 = 35°$
$d_1 = 3.0$ cm
$d_2 = d_3 = 2.0$ cm



REFERENCES

R.A. Ford, Homemade Lightning: Creative Experiments in Electricity 3rd edition, McGraw-Hill, 2001).

APPENDIX:
Experimental equipment for electrostatics. You can find interesting explanation of equipment how it works, in Wikipedia.

1. Electroscope

https://en.wikipedia.org/wiki/Electroscope

2. **Electrophorous**
   https://en.wikipedia.org/wiki/Electrophorus
3. **Faraday cage**
   https://en.wikipedia.org/wiki/Faraday_cage
4. **Van der Graaf generator**
   https://en.wikipedia.org/wiki/Van_de_Graaff_generator
5. **Leyden jar**
   https://en.wikipedia.org/wiki/Leyden_jar

APPENDIX-II

Submultiples		
Value	**SI symbol**	**Name**
10^{-1} C	dC	decicoulomb
10^{-2} C	cC	centicoulomb
10^{-3} C	**mC**	**millicoulomb**
10^{-6} C	**μC**	**microcoulomb**
10^{-9} C	**nC**	**nanocoulomb**
10^{-12} C	**pC**	**picocoulomb**
10^{-15} C	fC	femtocoulomb
10^{-18} C	aC	attocoulomb
10^{-21} C	zC	zeptocoulomb
10^{-24} C	yC	yoctocoulomb

Electric field

Electric field

المجال الكهربي

المحاضرة الرابعة

في هذا الفصل سنقوم بإدخال مفهوم المجال الكهربي الناشئ عن الشحنة أو الشحنات الكهربية، والمجال الكهربي هو ... ية والذي تظهر فيه تأثير القوى الكهربية. كذلك سندرس ... شحنة في حالة أن كون السرعة الابتدائية تساوي صفراً وكذلك في حالة شحنة متحركة.

3.1 The Electric Field

The gravitational field g at a point in space was defined to be equal to the gravitational force F acting on a test mass m_o divided by the test mass

$$\vec{g} = \frac{\vec{F}}{m_o} \qquad (3.1)$$

In the same manner, an ***electric field*** at a point in space can be defined in term of electric force acting on a test charge q_o placed at that point.

3.2 Definition of the electric field

The electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}$ acting on a positive test charge placed at that point divided by the magnitude of the test charge q_o

$$\vec{E} = \frac{\vec{F}}{q_o} \qquad (3.2)$$

The electric field has a unit of N/C

لاحظ هنا أن المجال الكهربي $\vec{E}$ هو مجال خارجي وليس المجال الناشئ من الشحنة q_o كما هو موضح في الشكل 3.1، وقد يكون هناك مجال كهربي عند أية نقطة في الفراغ بوجود أو عدم وجود الشحنة q_o ولكن وضع الشحنة q_o عند أية نقطة في الفراغ هو وسيلة لحساب المجال الكهربي من خلال القوى الكهربية المؤثرة عليها.

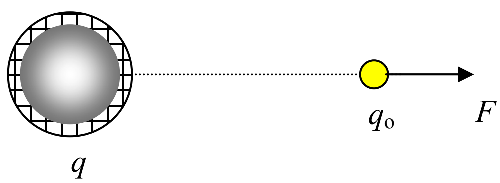


Figure 3.1

3.3 The direction of $\vec{E}$

If Q is +ve the electric field at point p in space is radially outward from Q as shown in figure 3.2(a).

If Q is -ve the electric field at point p in space is radially inward toward Q as shown in figure 3.2(b).

Figure 3.2 (a) ***Figure 3.2 (b)***

يكون اتجاه المجال عند نقطة ما لشحنة موجبة في اتجاه الخروج من النقطة كما في الشكل 3.2(a)، ويكون اتجاه المجال عند نقطة ما لشحنة سالبة في اتجاه الدخول من النقطة إلى الشحنة كما في الشكل 3.2(b).

3.4 Calculating $\vec{E}$ due to a charged particle

Consider Fig. 3.2(a) above, the magnitude of force acting on q_o is given by Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Qq_o}{r^2}$$

$$E = \frac{F}{q_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad (3.3)$$

3.5 To find *E* for a group of point charge

To find the magnitude and direction of the electric field due to several charged particles as shown in figure 3.3 use the following steps

(1) نرقم الشحنات المراد إيجاد المجال الكهربي لها.

(2) نحدد اتجاه المجال الكهربي لكل شحنة على حده عند النقطة المراد إيجاد محصلة المجال عندها ولتكن النقطة p، يكون اتجاه المجال خارجاً من النقطة p إذا كانت الشحنة موجبة ويكون اتجاه المجال داخلاً إلى النقطة إذا كانت الشحنة سالبة كما هو الحال في الشحنة رقم (2).

(3) يكون المجال الكهربي الكلي هو الجمع الاتجاهي لمتجهات المجال

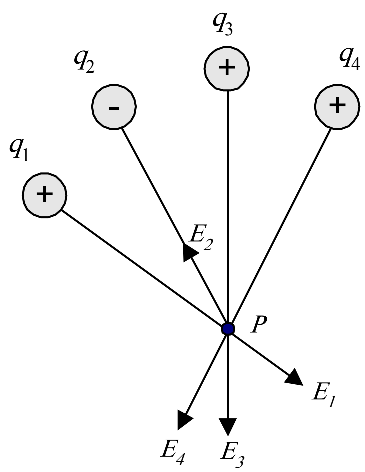


Figure 3.3

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \dots\dots\dots \qquad (3.4)$$

(4) إذا كان لا يجمع متجهات المجال خط عمل واحد نحلل كل متجه إلى مركبتين في اتجاه محوري x و y

(5) نجمع مركبات المحور x على حده ومركبات المحور y.

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x}$$
$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y}$$

(6) تكون قيمة المجال الكهربي عند النقطة p هي $E = \sqrt{E_x^2 + E_y^2}$

(7) يكون اتجاه المجال هو $\theta = \tan^{-1} \dfrac{E_y}{E_x}$

Example 3.1

Find the electric field at point *p* in figure 3.4 due to the charges shown.

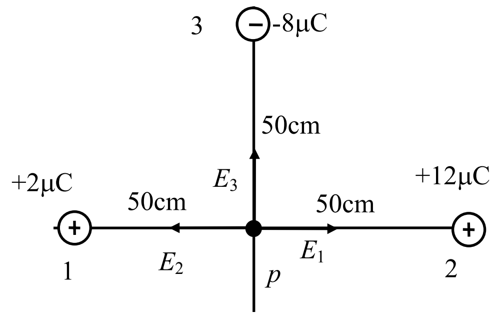


Figure 3.4

Solution

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_x = E_1 - E_2 = -36\times10^4 \text{N/C}$$

$$E_y = E_3 = 28.8\times10^4 \text{N/C}$$

$$E_p = \sqrt{(36\times10^4)^2 + (28.8\times10^4)^2} = 46.1 \text{N/C}$$

$$\theta = 141^\circ$$

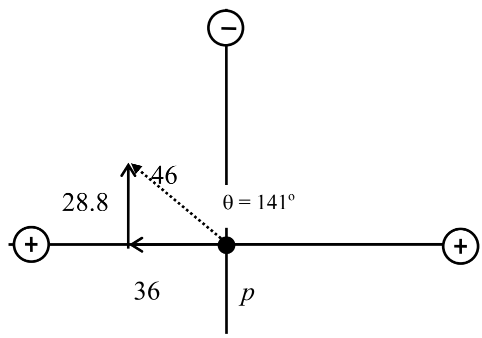


Figure 3.5 Shows the resultant electric field

Example 3.2

Find the electric field due to electric dipole along x-axis at point *p*, which is a distance *r* from the origin, then assume *r>>a*

***The electric dipole* is positive charge and negative charge of equal magnitude placed a distance 2*a* apart as shown in figure 3.6**

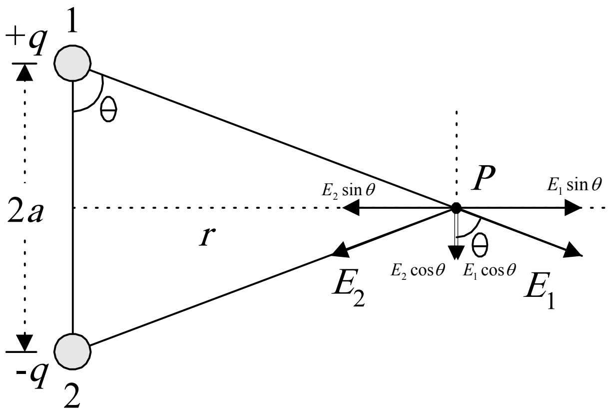


Figure 3.6

Solution

المجال الكلي عند النقطة p هو محصلة المجالين E_1 الناتج عن الشحنة q_1 والمجال E_2 الناتج عن الشحنة q_2 أي أن

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

وحيث أن النقطة p تبعد عن الشحنتين بنفس المقدار، والشحنتان متساويتان إذاً المجالان متساويان وقيمة المجال تعطى بالعلاقة

$$E_1 = \frac{1}{4\pi\varepsilon_o} \frac{q_1}{a^2 + r^2} = E_2$$

لاحظ هنا أن المسافة الفاصلة هي ما بين الشحنة والنقطة المراد إيجاد المجال عندها.

نحلل متجه المجال إلى مركبتين كما في الشكل أعلاه

$$E_x = E_1 \sin\theta - E_2 \sin\theta$$

$$E_y = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$E_p = 2E_1 \cos\theta$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \cos\theta$$

from the Figure

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{1}{4\pi\varepsilon_o} \frac{q}{a^2 + r^2} \frac{a}{\sqrt{a^2 + r^2}}$$

$$E_p = \frac{2aq}{4\pi\varepsilon_o (r^2 + a^2)^{3/2}} \qquad (3.5)$$

The direction of the electric field in the -ve y-axis.

The quantity $2aq$ is called the ***electric dipole momentum*** (P) and has a direction from the -ve charge to the +ve charge

(b) when $r >> a$

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o r^3} \qquad (3.6)$$

يتضح مما سبق أن المجال الكهربي الناشئ عن electric dipole عند نقطة واقعة على العمود المنصف بين الشحنتين يكون اتجاهه في عكس اتجاه electric dipole momentum وبالنسبة للنقطة البعيدة عن electric dipole فإن المجال يتناسب عكسيا مع مكعب المسافة، وهذا يعنى أن تناقص المجال مع المسافة يكون أكبر منه في حالة شحنة واحدة فقط.

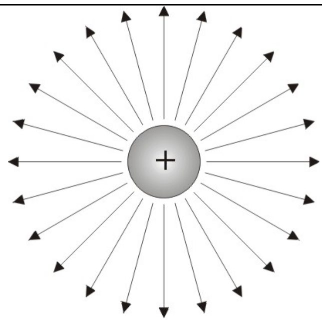
3.6 Electric field lines

The electric lines are a convenient way to visualize the electric filed patterns. The relation between the electric field lines and the electric field vector is this:

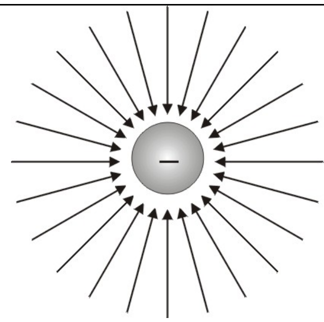
(1) The tangent to a line of force at any point gives the direction of $\vec{E}$ at that point.

(2) The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the magnitude of $\vec{E}$.

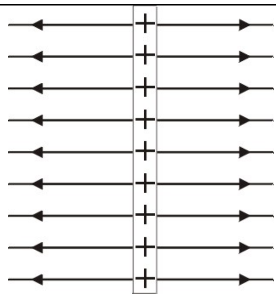
Some examples of electric line of force



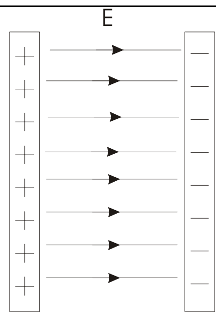
Electric field lines due to +ve charge



Electric field lines due to -ve charge

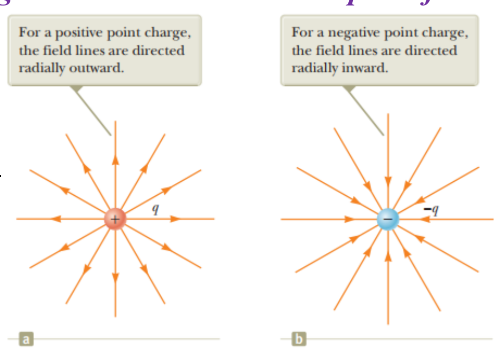


Electric field lines due to +ve line charge



Electric field lines due two surface charge

Figure 3.7 shows some examples of electric line of force



Notice that the rule of drawing the line of force:-

(1) The lines must begin on positive charges and terminates on negative charges.

(2) The number of lines drawn is proportional to the magnitude of the charge.

(3) No two electric field lines can cross.

3.7 Motion of charge particles in a uniform electric field

If we are given a field $\vec{E}$, what forces will act on a charge placed in it?

We start with special case of a point charge in uniform electric field $\vec{E}$. The electric field will exert a force on a charged particle is given by

$$F = qE$$

The force will produce acceleration

$$a = F/m$$

where m is the mass of the particle. Then we can write

$$F = qE = ma$$

The acceleration of the particle is therefore given by

$$a = qE/m \qquad (3.7)$$

If the charge is positive, the acceleration will be in the direction of the electric field. If the charge is negative, the acceleration will be in the direction opposite the electric field.

One of the practical applications of this subject is a device called the (***Oscilloscope***) See appendix A (***Cathode Ray Oscilloscope***) for further information.

3. A very small ball has a mass of 5.00×10^{-3} kg and a charge of 4.00 μC. What magnitude electric field directed upward will balance the weight of the ball so that the ball is suspended motionless above the ground? (a) 8.21×10^{2} N/C (b) 1.22×10^{4} N/C (c) 2.00×10^{-2} N/C (d) 5.11×10^{6} N/C (e) 3.72×10^{3} N/C

5. A point charge of -4.00 nC is located at (0, 1.00) m. What is the x component of the electric field due to the point charge at (4.00, -2.00) m? (a) 1.15 N/C (b) -0.864 N/C (c) 1.44 N/C (d) -1.15 N/C (e) 0.864 N/C

Two point charges attract each other with an electric force of magnitude F. If the charge on one of the particles is reduced to one-third its original value and the distance between the particles is doubled, what is the resulting magnitude of the electric force between them? (a) $\frac{1}{12}F$ (b) $\frac{1}{3}F$ (c) $\frac{1}{6}F$ (d) $\frac{3}{4}F$ (e) $\frac{3}{2}F$

49. Figure P23.49 shows the electric **W** field lines for two charged particles separated by a small distance. (a) Determine the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?

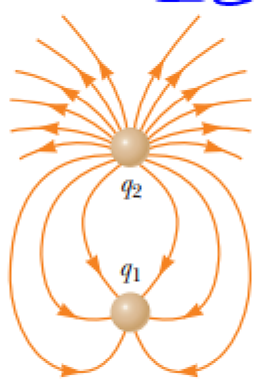


Figure P23.49

3.8 Solution of some selected problems

Example 3.3

A positive point charge q of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x-axis as shown in figure 3.8, describe its motion.

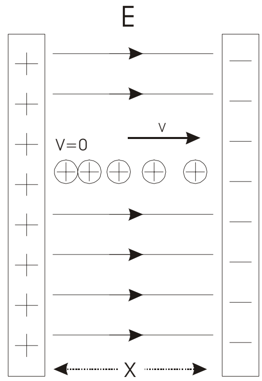


Figure 3.8

Solution

The acceleration is given by

$$a = qE/m$$

Since the motion of the particle in one dimension, then we can apply the equations of kinematics in one dimension

$$x-x_o= v_0t+ \tfrac{1}{2}\, at^2 \qquad v = v_0 + at \qquad v^2=v_o^{\,2} + 2a(x-x_o)$$

Taking $x_o = 0$ and $v_0 = 0$

$$x = \tfrac{1}{2}\, at^2 = (qE/2m)\, t^2$$

$$v = at = (qE/m)\, t$$

$$v^2 =2ax = (2qE/m)x \qquad (3.7)$$

Example 3.4

In the above example suppose that a negative charged particle is projected horizontally into the uniform field with an initial velocity v_o as shown in figure 3.9.

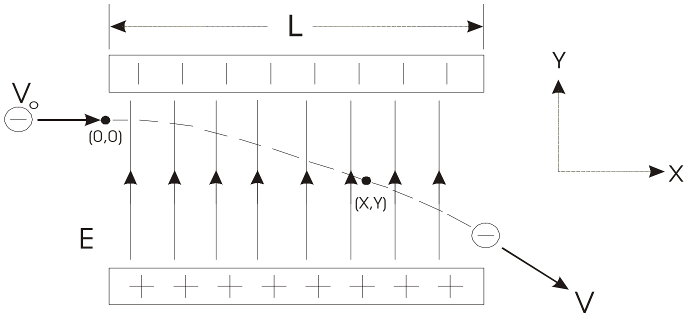


Figure 3.9

Solution

Since the direction of electric field $\vec{E}$ in the y direction, and the charge is negative, then the acceleration of charge is in the direction of -y.

$$a = -qE/m$$

The motion of the charge is in two dimension with constant acceleration, with $v_{xo} = v_o$ & $v_{yo} = 0$

The components of velocity after time t are given by

$$v_x = v_o = \text{constant}$$

$$v_y = at = -(qE/m)\,t$$

The coordinate of the charge after time t are given by

$$x = v_o t$$

$$y = \tfrac{1}{2} at^2 = -1/2\,(qE/m)\,t^2$$

Eliminating t we get

$$y = \frac{qE}{2mv_0^2}x^2 \qquad (3.8)$$

we see that y is proportional to x^2. Hence, the trajectory is parabola.

Example 3.5

Find the electric field due to electric dipole shown in figure 3.10 along x-axis at point *p* which is a distance *r* from the origin. then assume *r>>a*

Solution

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2$$

$$E_1 = K\frac{q}{(x+a)^2}$$

$$E_2 = K\frac{q}{(x-a)^2}$$

$$E_p = K\,\frac{q}{(x-a)^2} - \frac{q}{(x+a)^2}$$

$$E_p = Kq\frac{4ax}{(x^2-a^2)^2}$$

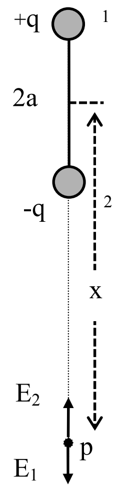


Figure 3.10

When $x>>a$ then

$$\therefore E = \frac{2aq}{4\pi\varepsilon_o x^3} \qquad (3.9)$$

لاحظ الإجابة النهائية عندما تكون x أكبر كثيرا من المسافة $2a$ حيث يتناسب المجال عكسيا مع مكعب المسافة.

Example 3.6

What is the electric field in the lower left corner of the square as shown in figure 3.11? Assume that $q = 1\times10^{-7}$C and a = 5cm.

Solution

First we assign number to the charges (1, 2, 3, 4) and then determine the direction of the electric field at the point p due to the charges.

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{2a^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o}\frac{2q}{a^2}$$

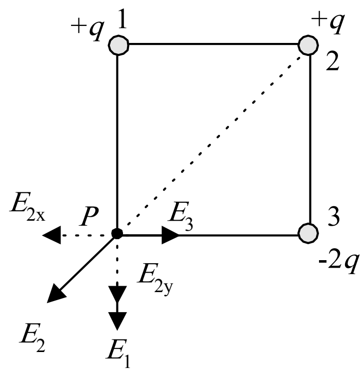


Figure 3.11

Evaluate the value of E_1, E_2, & E_3

$E_1 = 3.6\times10^5$ N/C,

$E_2 = 1.8 \times 10^5$ N/C,

$E_3 = 7.2 \times 10^5$ N/C

Since the resultant electric field is the vector additions of all the fields *i.e.*

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

We find the vector E_2 need analysis to two components

$E_{2x} = E_2 \cos 45$

$E_{2y} = E_2 \sin 45$

$E_x = E_3 - E_2 \cos 45 = 7.2\times10^5 - 1.8 \times 10^5 \cos 45 = 6 \times 10^5$ N/C

$$E_y = -E_1 - E_2 \sin 45 = -3.6\times10^5 - 1.8 \times 10^5 \sin 45 = -4.8 \times 10^5 \text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} \qquad = 7.7 \times 10^5 \text{ N/C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} \qquad = -38.6^{\circ}$$

Example 3.7

In figure 3.12 shown, locate the point at which the electric field is zero? Assume *a* = 50cm

Solution

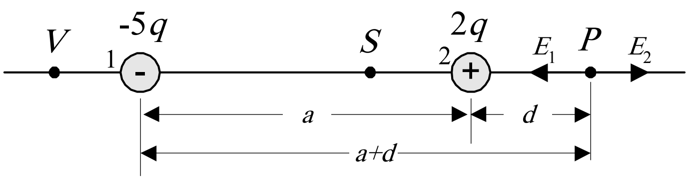


Figure 3.12

To locate the points at which the electric field is zero (E=0), we shall try all the possibilities, assume the points S, V, P and find the direction of E_1 and E_2 at each point due to the charges q_1 and q_2.

The resultant electric field is zero only when E_1 and E_2 are equal in magnitude and opposite in direction.

At the point S E_1 in the same direction of E_2 therefore E cannot be zero in between the two charges.

At the point V the direction of E_1 is opposite to the direction of E_2, but the magnitude could not be equal (can you find the reason?)

At the point P the direction of E_1 and E_2 are in opposite to each other and the magnitude can be equal

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o}\frac{2q}{(0.5+d)^2} = \frac{1}{4\pi\varepsilon_o}\frac{5q}{(d)^2}$$

$$d = 30\text{cm}$$

لاحظ هنا أنه في حالة الشحنتين المتشابهتين فإن النقطة التي ينعدم عندها المجال تكون بين الشحنتين، أما إذا كانت الشحنتان مختلفتين في الإشارة فإنها تكون خارج إحدى الشحنتين وعلى الخط الواصل بينهما وبالقرب من الشحنة الأصغر.

Example 3.8

A charged cord ball of mass 1g is suspended on a light string in the presence of a uniform electric field as in figure 3.13. When $E=(3i+5j)\times10^5$N/C, the ball is in equilibrium at $\theta=37^o$. Find (a) the charge on the ball and (b) the tension in the string.

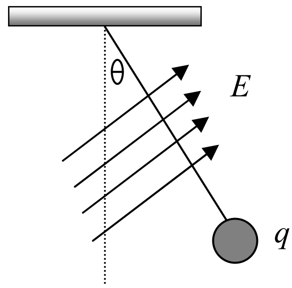
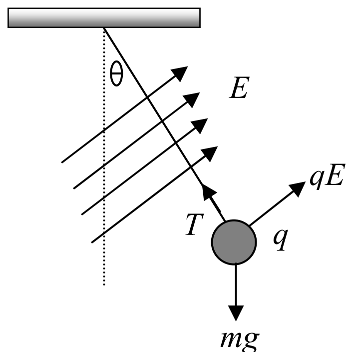


Figure 3.13

Solution



حيث أن الكرة مشحونة بشحنة موجبة فإن القوة الكهربية المؤثرة على الكرة المشحونة في اتجاه المجال الكهربي.

كما أن الكرة المشحونة في حالة اتزان فإن محصلة القوى المؤثرة على الكرة ستكون صفر. بتطبيق قانون نيوتن الثاني $\sum F=ma$ على مركبات x و y.

$E_x = 3\times10^5$N/C $\qquad$ $E_y = 5j\times10^5$N/C

$\sum F = T+qE+F_g = 0$

$\sum F_x = qE_x - T \sin 37 = 0 \qquad (1)$

$\sum F_y = qE_y + T \cos 37 - mg = 0 \qquad (2)$

Substitute T from equation (1) into equation (2)

$$q = \frac{mg}{\left(E_y + \frac{E_x}{\tan 37}\right)} = \frac{(1\times10^{-3})(9.8)}{\left(5+\frac{3}{\tan 37}\right)\times10^5} = 1.09\times10^{-8}\ C$$

To find the tension we substitute for q in equation (1)

$$T = \frac{qEx}{\sin 37} = 5.44\times10^{-3}\,N$$

3.9 The electric dipole in electric field

If an electric dipole placed in an external electric field *E* as shown in figure 3.14, then a torque will act to align it with the direction of the field.

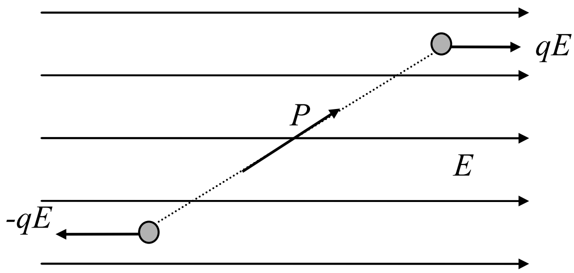


Figure 3.14

$$\vec{\tau} = \vec{P} \times \vec{E} \qquad (3.10)$$

$$\tau = P\,E \sin\theta \qquad (3.11)$$

where *P* is the electric dipole momentum, θ the angle between *P* and *E*

يكون ثنائي القطب في حالة اتزان equilibrium عندما يكون الازدواج مساويا للصفر وهذا يتحقق عندما تكون ($\theta = 0 \ , \ \pi$)

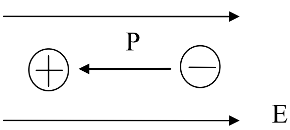


Figure 3.15 (ii)

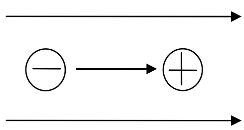


Figure 3.15 (i)

في الوضع الموضح في الشكل 3.15(i) عندما $\theta = 0$ يقال إن الـ dipole في وضع اتزان مستقر stable equilibrium لأنه إذا أزيح بزاوية صغيرة فانه سيرجع إلى الوضع $\theta = 0$، بينما في الوضع الموضح في الشكل 3.15(ii) يقال إن الـ dipole في وضع اتزان غير مستقر unstable equilibrium لأن إزاحة صغيرة له سوف تعمل على أن يدور الـ dipole ويرجع إلى الوضع $\theta = 0$ وليس $\theta = \pi$.

3.10 Problems

3.1) The electric force on a point charge of 4.0μC at some point is 6.9×10^{-4}N in the positive x direction. What is the value of the electric field at that point?

3.2) What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table l.)

3.3) A point charge of -5.2μC is located at the origin. Find the electric field (a) on the x-axis at x=3 m, (b) on the y-axis at y= -4m, (c) at the point with coordinates x=2m, y=2m.

3.4) What is the magnitude of a point charge chosen so that the electric field 50cm away has the magnitude 2.0N/C?

3.5) Two point charges of magnitude $+2.0\times10^{-7}$C and $+8.5\times10^{-11}$C are 12cm apart. (a) What electric field does each produce at the site of the other? (b) What force acts on each?

3.6) An electron and a proton are each placed at rest in an external electric field of 520N/C. Calculate the speed of each particle after 48nanoseconds.

3.7) The electrons in a particle beam each have a kinetic energy of 1.6×10^{-17}J. What are the magnitude and direction of the electric field that will stop these electrons in a distance of 10cm?

3.8) A particle having a charge of -2.0×10^{-9}C is acted on by a downward electric force of 3.0×10^{-6}N in a uniform electric field. (a) What is the strength of the electric field? (b) What is the magnitude and direction of the electric force exerted on a proton placed in this field? (c) What is the gravitational force on the proton? (d) What is the ratio of the electric to the gravitational forces in this case?

3.9) Find the total electric field along the line of the two charges shown in figure 3.16 at the point midway between them.

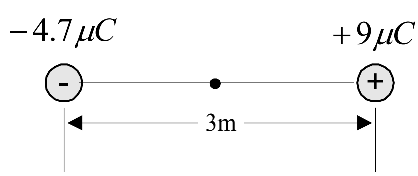


Figure 3.16

3.10) What is the magnitude and direction of an electric field that will balance the weight of (a) an electron and (b) a proton?

3.11) Three charges are arranged in an equilateral triangle as shown in figure 3.17. What is the direction of the force on $+q$?

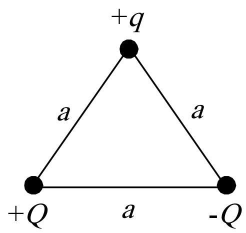


Figure 3.17

3.12) In figure 3.18 locate the point at which the electric field is zero and also the point at which the electric potential is zero. Take q=1μC and a=50cm.

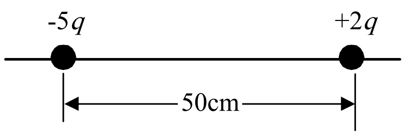


Figure 3.18

3.13) What is E in magnitude and direction at the center of the square shown in figure 3.19? Assume that q=1μC and a=5cm.

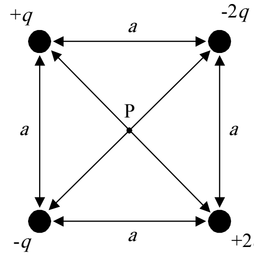


Figure 3.19

3.14) Two point charges are a distance d apart (Figure 3.20). Plot $E(x)$, assuming x=0 at the left-hand charge. Consider both positive and negative values of x. Plot E as positive if E points to the right and negative if E points to the left. Assume $q_1=+1.0\times10^{-6}$C, $q_2=+3.0\times10^{-6}$C, and d=10cm.

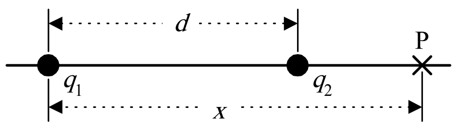


Figure 3.20

3.15) Calculate E (direction and magnitude) at point P in Figure 3.21.

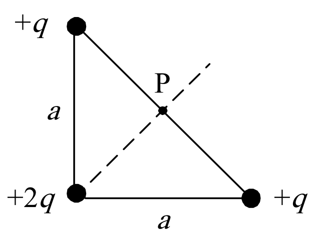


Figure 3.21

3.16) Charges $+q$ and $-2q$ are fixed a distance d apart as shown in figure 3.22. Find the electric field at points A, B, and C.

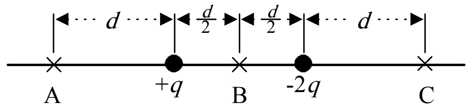


Figure 3.22

3.17) A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0cm away, in a time 1.5×10^{-8}s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field E?

Electric Flux

Electric Flux
التدفق الكهربي

المحاضرة السادسة

درسنا سابقا كيفية حساب المجال لتوزيع معين من الشحنات باستخدام قانون كولوم. وهنا سنقدم طريقة أخرى لحساب المجال الكهربي باستخدام "قانون جاوس" الذي يسهل حساب المجال الكهربي لتوزيع متصل من الشحنة على شكل توزيع طولي أو سطحي أو حجمي. يعتمد قانون جاوس ... كهربي الناتج من المجال الكهربي أو الشحنة الكهربائية، ولهذا سنقوم اولا بحساب التدفق الكهربي الناتج عن المجال الكهربي، وثانياً سنقوم بحساب التدفق الكهربي الناتج عن شحنة كهربية، ومن ثم سنقوم بإيجاد قانون جاوس واستخدامه في بعض التطبيقات الهامة في مجال الكهربية الساكنة.

4.1 The Electric Flux due to an Electric Field

We have already shown how electric field can be described by lines of force. A line of force is an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point. Field lines never intersect, since only one line can pass through a single point.

The Electric flux (Φ) is a measure of the number of electric field lines penetrating some surface of area A.

Case one:

The electric flux for a plan surface perpendicular to a uniform electric field (figure 4.1)

To calculate the electric flux we recall that the number of lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of lines penetrating the surface of area A is proportional to the product EA. The product of the electric filed E and the surface area A perpendicular to the field is called the electric flux Φ.

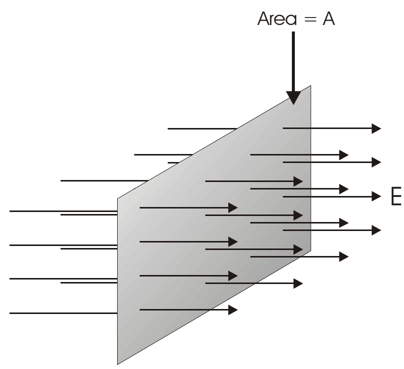


Figure 4.1

$$\Phi = \vec{E}.\vec{A} \tag{4.1}$$

The electric flux Φ has a unit of $N.m^2/C$.

Case Two

The electric flux for a plan surface make an angle θ to a uniform electric field (figure 4.2)

Note that the number of lines that cross-area is equal to the number that cross the projected area $A`$, which is perpendicular to the field. From the figure we see that the two area are related by $A` = A\cos\theta$. The flux is given by:

$$\Phi = \vec{E}.\vec{A}' = E\,A\,cos\theta$$

$$\Phi = \vec{E}.\vec{A}$$

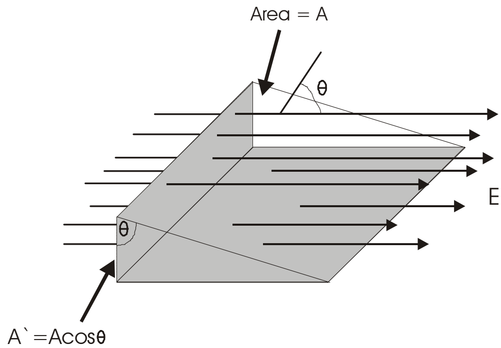


Figure 4.2

Where θ is the angle between the electric field E and the normal to the surface $\vec{A}$.

إذاً يكون الفيض ذا قيمة عظمى عندما يكون السطح عمودياً على المجال أي $\theta = 0$ ويكون ذا قيمة صغرى عندما يكون السطح موازياً للمجال أي عندما $\theta = 90$. لاحظ هنا أن المتجه $\vec{A}$ هو متجه المساحة وهو عمودي دائما على المساحة وطوله يعبر عن مقدار المساحة.

Case Three

In general the electric field is nonuniform over the surface (figure 4.3)

The flux is calculated by integrating the normal component of the field over the surface in question.

$$\Phi = \oint \vec{E}.\vec{A} \qquad (4.2)$$

The ***net flux*** through the surface is proportional to the ***net number of lines*** penetrating the surface

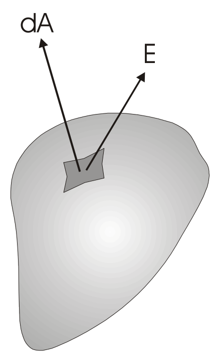


Figure 4.3

والمقصود بـ net number of lines أي عدد الخطوط الخارجة من السطح (إذا كانت الشحنة موجبة) - عدد الخطوط الداخلة إلى السطح (إذا كانت الشحنة سالبة).

Example 4.1

What is electric flux Φ for closed cylinder of radius *R* immersed in a uniform electric field as shown in figure 4.4?

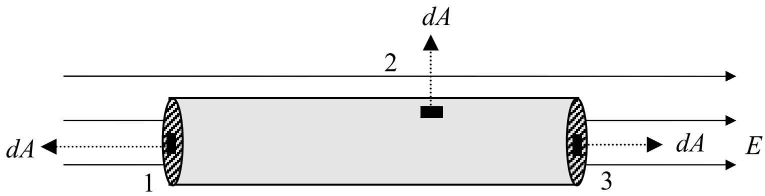


Figure 4.4

Solution

نطبق قانون جاوس على الأسطح الثلاثة الموضحة في الشكل أعلاه

$$\Phi = \oint \vec{E}.d\vec{A} = \oint_{(1)} \vec{E}.d\vec{A} + \oint_{(2)} \vec{E}.d\vec{A} + \oint_{(3)} \vec{E}.d\vec{A}$$

$$= \oint_{(1)} E\cos 180 dA + \oint_{(2)} E\cos 90 dA + \oint_{(3)} E\cos 0 dA$$

Since E is constant then

$$\Phi = - EA + 0 + EA = \text{zero}$$

Exercise

Calculate the total flux for a cube immersed in uniform electric field $\vec{E}$.

4.2 The Electric Flux due to a point charge

To calculate the electric flux due to a point charge we consider an imaginary closed spherical surface with the point charge in the center figure 4.5, this surface is called ***gaussian surface.*** Then the flux is given by

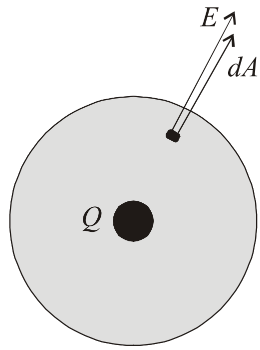


Figure 4.5

$$\Phi = \oint \vec{E}.d\vec{A} = E\oint dA\cos\theta \qquad (\theta = 0)$$

$$\Phi = \frac{q}{4\pi\varepsilon_o r^2}\int dA = \frac{q}{4\pi\varepsilon_o r^2}4\pi r^2$$

$$\Phi = \frac{q}{\varepsilon_o} \qquad (4.3)$$

Note that the net flux through a spherical gaussian surface is proportional to the charge q inside the surface.

4.3 Gaussian surface

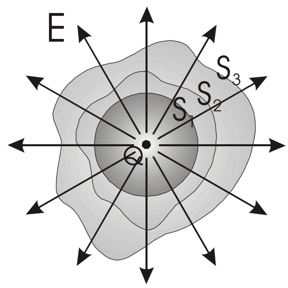


Figure 4.6

Consider several closed surfaces as shown in figure 4.6 surrounding a charge Q as in the figure below. The flux that passes through surfaces S_1, S_2 and S_3 all has a value q/ε_o. Therefore we conclude that the net flux through any closed surface is independent of the shape of the surface.

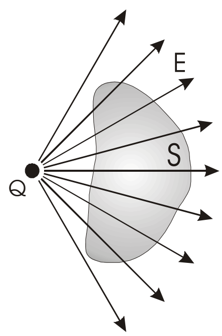


Figure 4.7

Consider a point charge located outside a closed surface as shown in figure 4.7. We can see that the number of electric field lines entering the surface equal the number leaving the surface. Therefore the net electric flux in this case is zero, because the surface surrounds no electric charge.

Example 4.2

In figure 4.8 two equal and opposite charges of $2Q$ and $-2Q$ what is the flux Φ for the surfaces S_1, S_2, S_3 and S_4.

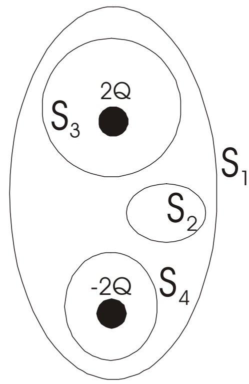


Figure 4.8

Solution

For S_1 the flux Φ = zero

For S_2 the flux Φ = zero

For S_3 the flux $\Phi = +2Q/\varepsilon_o$

For S_4 the flux $\Phi = -2Q/\varepsilon_o$

4.4 Gauss's Law

Gauss law is a very powerful theorem, which relates any charge distribution to the resulting electric field at any point in the vicinity of the charge. As we saw the electric field lines means that each charge q must have q/ε_o flux lines coming from it. This is the basis for an important equation referred to as ***Gauss's law***. Note the following facts:

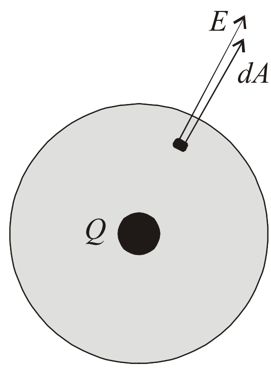


Figure 4.9

1. If there are charges q_1, q_2, q_3, ...q_n inside a closed (gaussian) surface, the total number of flux lines coming from these charges will be

$$(q_1 + q_2 + q_3 + ... + q_n)/\varepsilon_o \qquad (4.4)$$

2. The number of flux lines coming out of a closed surface is the integral of $\vec{E}.d\vec{A}$ over the surface, $\oint \vec{E}.d\vec{A}$

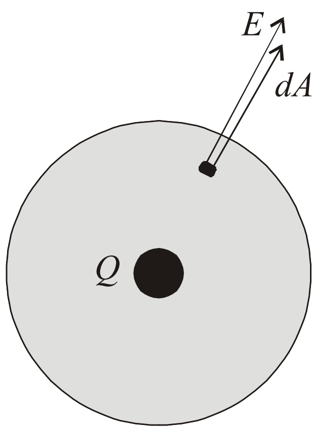
We can equate both equations to get Gauss law which state that the net electric flux through a closed gaussian surface is equal to the net charge inside the surface divided by ε_o

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \qquad \textbf{Gauss's law} \qquad (4.5)$$

where q_{in} is the total charge inside the gaussian surface.

Gauss's law states that the net electric flux through any closed gaussian surface is equal to the net electric charge inside the surface divided by the permittivity.

4.5 Gauss's law and Coulomb's law



We can deduce Coulomb's law from Gauss's law by assuming a point charge q, to find the electric field at point or points a distance r from the charge we imagine a spherical gaussian surface of radius r and the charge q at its center as shown in figure 4.10.

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$\oint E\cos 0 dA = \frac{q_{in}}{\varepsilon_o}$$

Because E is constant for all points on the sphere, it can be factored from the inside of the integral sign, then

$$E\oint dA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad EA = \frac{q_{in}}{\varepsilon_o} \quad \Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (4.6)$$

Now put a second point charge q_o at the point, which E is calculated. The magnitude of the electric force that acts on it $F = Eq_o$

$$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

4.6 Conductors in electrostatic equilibrium

A good electrical conductor, such as copper, contains charges (electrons) that are free to move within the material. When there is no net motion of charges within the conductor, the conductor is in electrostatic equilibrium.

Conductor in electrostatic equilibrium has the following properties:

1. Any excess charge on an isolated conductor must reside entirely on its surface. *(Explain why?) The answer is when an **excess charge** is placed on a conductor, it will set-up electric field inside the conductor. These fields act on the charge carriers of the conductor (electrons) and cause them to move i.e. current flow inside the conductor. These currents redistribute the **excess charge** on the surface in such away that the internal electric fields reduced to become zero and the currents stop, and the electrostatic conditions restore.*

2. The electric field is zero everywhere inside the conductor. *(Explain why?) Same reason as above*

In figure 4.11 it shows a conducting slab in an external electric field E. The charges induced on the surface of the slab produce an electric field, which opposes the external field, giving a resultant field of zero in the conductor.

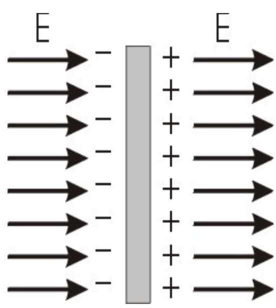


Figure 4.11

المحاضرة السابعة

Steps which should be followed in solving problems

1. The gaussian surface should be chosen to have the same symmetry as the charge distribution.

2. The dimensions of the surface must be such that the surface includes the point where the electric field is to be calculated.

3. From the symmetry of the charge distribution, determine the direction of the electric field and the surface area vector dA, over the region of the gaussian surface.

4. Write $E.dA$ as $E\ dA\ cos\theta$ and divide the surface into separate regions if necessary.

5. The total charge enclosed by the gaussian surface is $dq = \int dq$, which is represented in terms of the charge density ($dq = \lambda dx$ for line of charge, $dq = \sigma dA$ for a surface of charge, $dq = \rho dv$ for a volume of charge).

4.7 Applications of Gauss's law

كما ذكرنا سابقا فإن قانون جاوس يطبق على توزيع متصل من الشحنة، وهذا التوزيع إما أن يكون توزيعاً طولياً أو توزيعاً سطحياً أو توزيعاً حجمياً. يوجد على كل حالة مثال محلول في الكتاب سنكتفي هنا بذكر بعض النقاط الهامة.

على سبيل المثال إذا أردنا حساب المجال الكهربي عند نقطة تبعد مسافة عن سلك مشحون كما في الشكل 4.12، هنا في هذه الحالة الشحنة موزعة بطريقة متصلة، وغالبا نفترض أن توزيع الشحنة منتظم ويعطى بكثافة التوزيع λ(C/m)، ولحل مثل هذه المشكلة نقسم السلك إلى عناصر صغيرة طول كلا منها dx ونحسب المجال dE الناشئ عند نقطة (p)

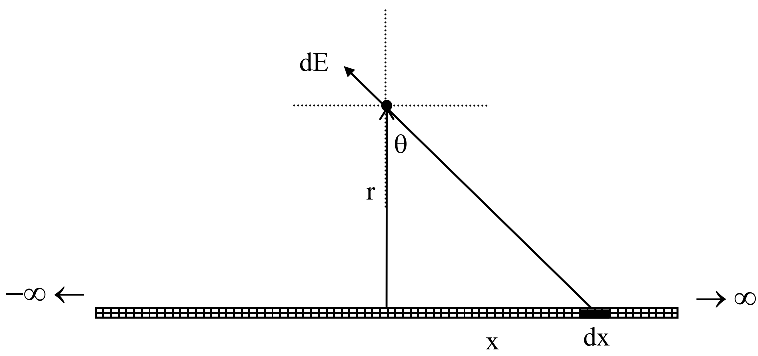


Figure 4.12

$$dE = K\frac{dq}{r^2+x^2} = K\frac{\lambda dx}{r^2+x^2}$$

ومن التماثل نجد أن المركبات الأفقية تتلاشى والمحصلة تكون في اتجاه المركبة الرأسية التي في اتجاه y

$$dE_y = dE\cos\theta \qquad\qquad E_y = \int dE_y = \int_{-\infty}^{+\infty}\cos\theta\, dE$$

$$E = 2\int_{0}^{+\infty} \cos\theta dE \qquad \frac{2\lambda}{4\pi\varepsilon_o}\int_{0}^{+\infty} \cos\theta \frac{dx}{r^2 + x^2}$$

من الشكل الهندسي يمكن التعويض عن المتغير x والمتغير dx كما يلي:

$$x = y \tan\theta \qquad \Rightarrow \qquad dx = y \sec^2\theta \, d\theta$$

$$E = \frac{\lambda}{2\pi\varepsilon_o}\int_{0}^{\pi/2} \cos\theta d\theta$$

انتبه إلى حدود التكامل

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

لاشك أنك لاحظت صعوبة الحل باستخدام قانون كولوم في حالة التوزيع المتصل للشحنة، لذلك سندرس قانون جاوس الذي يسهل الحل كثيراً في مثل هذه الحالات والتي بها درجة عالية من التماثل.

Gauss's law can be used to calculate the electric field if the symmetry of the charge distribution is high. Here we concentrate in three different ways of charge distribution

	1	**2**	**3**
Charge distribution	Linear	Surface	Volume
Charge density	λ	σ	ρ
Unit	C/m	C/m^2	C/m^3

A linear charge distribution

In figure 4.13 calculate the electric field at a distance *r* from a uniform positive line charge of infinite length whose charge per unit length is λ=constant.

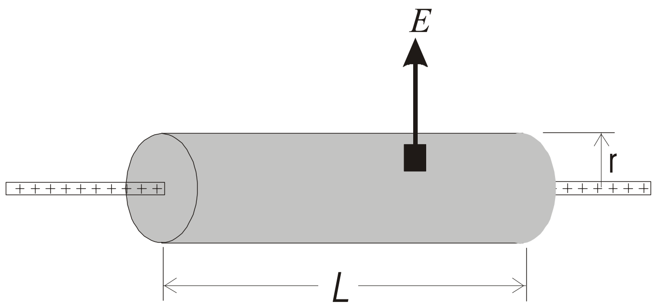


Figure 4.13

The electric field *E* is perpendicular to the line of charge and directed outward. Therefore for symmetry we select a cylindrical gaussian surface of radius r and length *L*.

The electric field is constant in magnitude and perpendicular to the surface.

The flux through the end of the gaussian cylinder is zero since *E* is parallel to the surface.

The total charge inside the gaussian surface is λ*L*.
Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint dA = \frac{\lambda L}{\varepsilon_o}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_o}$$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_o r} \qquad (4.7)$$

نلاحظ هنا أنه باستخدام قانون جاوس سنحصل على نفس النتيجة التي توصلنا لها بتطبيق قانون كولوم وبطريقة أسهل.

A surface charge distribution

In figure 4.4 calculate the electric field due to non-conducting, infinite plane with uniform charge per unit area σ.

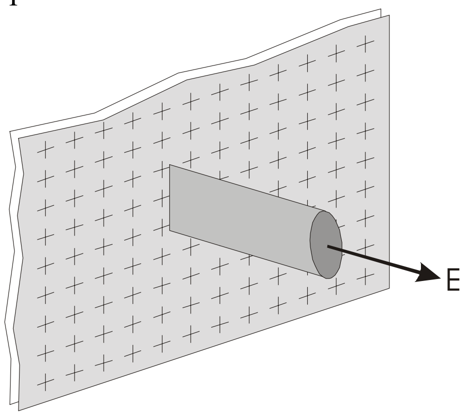


Figure 4.14

The electric field E is constant in magnitude and perpendicular to the plane charge and directed outward for both surfaces of the plane. Therefore for symmetry we select a cylindrical gaussian surface with its axis is perpendicular to the plane, each end of the gaussian surface has area A and are equidistance from the plane.

The flux through the end of the gaussian cylinder is EA since E is perpendicular to the surface.

The total electric flux from both ends of the gaussian surface will be $2EA$.
Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_o} \qquad (4.8)$$

An insulated conductor.

ذكرنا سابقا أن الشحنة توزع على سطح الموصل فقط، وبالتالي فإن قيمة المجال داخل مادة الموصل تساوى صفراً، وقيمة المجال خارج الموصل تساوى

$$E = \frac{\sigma}{\varepsilon_o} \qquad (4.9)$$

لاحظ هنا أن المجال في حالة الموصل يساوى ضعف قيمة المجال في حالة السطح اللانهائي المشحون، وذلك لأن خطوط المجال تخرج من السطحين في حالة السطح غير الموصل، بينما كل خطوط المجال تخرج من السطح الخارجي في حالة الموصل.

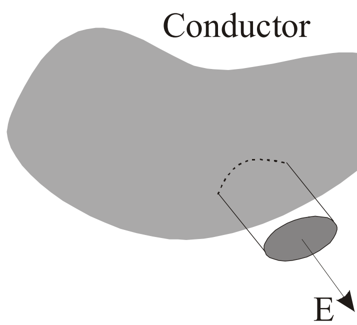


Figure 4.15

في الشكل الموضح أعلاه 4.15 نلاحظ أن الوجه الأمامي لسطح جاوس له فيض حيث أن الشحنة تستقر على السطح الخارجي، بينما يكون الفيض مساوياً للصفر للسطح الخلفي الذي يخترق الموصل وذلك لأن الشحنة داخل الموصل تساوي صفراً.

A volume charge distribution

In figure 4.16 shows an insulating sphere of radius *a* has a uniform charge density ρ and a total charge *Q*.

1) Find the electric field at point outside the sphere ($r>a$)

2) Find the electric field at point inside the sphere ($r<a$)

For $r>a$

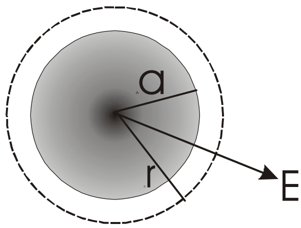


Figure 4.16

We select a spherical gaussian surface of radius *r*, concentric with the charge sphere where $r>a$. The electric field *E* is perpendicular to the gaussian surface as shown in figure 4.16. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\oint A = E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_o r^2} \qquad \text{(for r>a)} \qquad (4.10)$$

Note that the result is identical to appoint charge.

For *r<a*

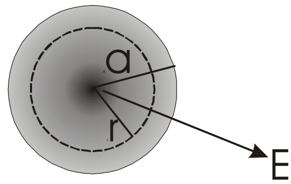


Figure 4.17

We select a spherical gaussian surface of radius r, concentric with the charge sphere where $r<a$. The electric field E is perpendicular to the gaussian surface as shown in figure 4.17. Applying Gauss law we get

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

It is important at this point to see that the charge inside the gaussian surface of volume $V`$ is less than the total charge $Q$. To calculate the charge $q_{in}$, we use $q_{in}=\rho V`$, where $V`=4/3\pi r^3$. Therefore,

$$q_{in} = \rho V` = \rho(4/3\pi r^3) \qquad (4.11)$$

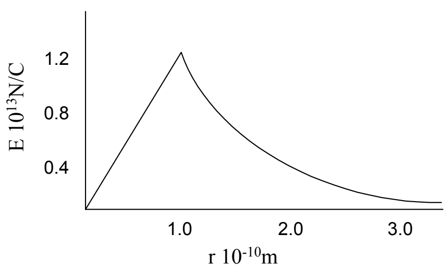
$$E\oint A = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_o}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_o r^2} = \frac{\rho}{3\varepsilon_o} r \qquad (4.12)$$

since $\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

$$\therefore E = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad \text{(for } r<a\text{)} \qquad (4.13)$$

Note that the electric field when $r<a$ is proportional to r, and when $r>a$ the electric field is proportional to $1/r^2$.



4.8 Solution of some selected problems

في هذا الجزء سنعرض حلولاً لبعض المسائل التي تغطى استخدام قانون جاوس
لإيجاد المجال الكهربي

4.8 Solution of some selected problems

Example 4.3

If the net flux through a gaussian surface is zero, which of the following statements are true?

1) **There are no charges inside the surface.**
2) **The net charge inside the surface is zero.**
3) **The electric field is zero everywhere on the surface.**
4) **The number of electric field lines entering the surface equals the number leaving the surface.**

Solution

Statements (b) and (d) are true. Statement (a) is not necessarily true since Gauss' Law says that the net flux through the closed surface equals the net charge inside the surface divided by ε_o. For example, you could have an electric dipole inside the surface. Although the net flux may be zero, we cannot conclude that the electric field is zero in that region.

Example 4.4

A spherical gaussian surface surrounds a point charge *q*. Describe what happens to the: flux through the surface if

1) **The charge is tripled,**
2) **The volume of the sphere is doubled,**
3) **The shape of the surface is changed to that of a cube,**
4) **The charge is moved to another position inside the surface;**

Solution

1) If the charge is tripled, the flux through the surface is tripled, since the net flux is proportional to the charge inside the surface
2) The flux remains unchanged when the volume changes, since it still surrounds the same amount of charge.
3) The flux does not change when the shape of the closed surface changes.

4) The flux through the closed surface remains unchanged as the charge inside the surface is moved to another position. All of these conclusions are arrived at through an understanding of Gauss' Law.

Example 4.5

A solid conducting sphere of radius a has a net charge $+2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$ as shown in figure 4.18. Using Gauss's law find the electric field in the regions labeled 1, 2, 3, 4 and find the charge distribution on the spherical shell.

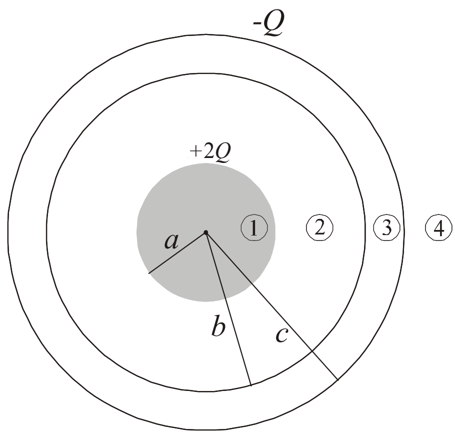


Figure 4.18

Solution

نلاحظ أن توزيع الشحنة على الكرتين لها تماثل كروي، لذلك لتعيين المجال الكهربي عند مناطق مختلفة فإننا سنفرض أن سطح جاوس كروي الشكل نصف قطره r.

Region (1) $r < a$

To find the E inside the solid sphere of radius a we construct a gaussian surface of radius $r < a$

$E = 0$ since no charge inside the gaussian surface.

Region (2) a $< r < b$

we construct a spherical gaussian surface of radius r

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

لاحظ هنا أن الشحنة المحصورة داخل سطح جاوس هي شحنة الكرة الموصلة الداخلية $2Q$ وأن خطوط المجال في اتجاه أنصاف الأقطار وخارجه من سطح جاوس أي $\theta = 0$ و المجال ثابت المقدار على السطح.

$$E\,4\pi r^2 = \frac{2Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2} \qquad a < r < b$$

Region (4) $r > c$

we construct a spherical gaussian surface of radius $r > c$, the total net charge inside the gaussian surface is $q = 2Q + (-Q) = +Q$ Therefore Gauss's law gives

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_o}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad r > c$$

Region (3) $b > r < c$

المجال الكهربي في هذه المنطقة يجب أن يكون صفراً لأن القشرة الكروية موصلة أيضا، ولأن الشحنة الكلية داخل سطح جاوس $b<r<c$ يجب أن تساوى صفراً. إذا نستنتج أن الشحنة $-Q$ على القشرة الكروية هي نتيجة توزيع شحنة على السطح الداخلي والسطح الخارجي للقشرة الكروية بحيث تكون المحصلة $-Q$ وبالتالي تتكون بالحث شحنة على السطح الداخلي للقشرة مساوية في المقدار للشحنة على الكرة الداخلية ومخالفة لها في الإشارة أي $-2Q$ وحيث أنه كما في معطيات السؤال الشحنة الكلية على القشرة الكروية هي $-Q$ نستنتج أن على السطح الخارجي للقشرة الكروية يجب أن تكون $+Q$

Example 4.6

A long straight wire is surrounded by a hollow cylinder whose axis coincides with that wire as shown in figure 4.19. The solid wire has a charge per unit length of +λ, and the hollow cylinder has a *net* charge per unit length of +2λ. Use Gauss law to find (a) the charge per unit length on the inner and outer surfaces of the hollow cylinder and (b) the electric field outside the hollow cylinder, a distance *r* from the axis.

Solution

(a) Use a cylindrical Gaussian surface S_1 within the conducting cylinder where E=0

Thus $$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} = 0$$

and the charge per unit length on the inner surface must be equal to

$$\lambda_{\text{inner}} = -\lambda$$

Also $$\lambda_{\text{inner}} + \lambda_{\text{outer}} = 2\lambda$$

thus $$\lambda_{\text{outer}} = 3\lambda$$

(b) For a gaussian surface S_2 outside the conducting cylinder

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o}$$

$$E\,(2\pi rL) = \frac{1}{\varepsilon_o}\,(\lambda - \lambda + 3\lambda)L$$

$$\therefore E = \frac{3\lambda}{2\pi\varepsilon_o r}$$

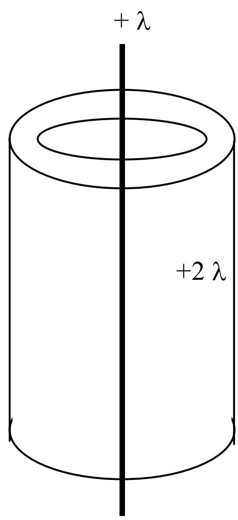


Figure 4.19

Example 4.7

Consder a long cylindrical charge distribution of radius *R* with a uniform charge density ρ. Find the electric field at distance *r* from the axis where *r*<*R*.

Solution

If we choose a cilindrical gaussian surface of length L and radius r, Its volume is $\pi r^2 L$, and it enclses a charge $\rho \pi r^2 L$. By applying Gauss's law we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_o} \quad \text{becomes} \qquad E\oint dA = \frac{\rho \pi r^2 L}{\varepsilon_o}$$

$$\because \oint dA = 2\pi r L \ \text{ therefore} \qquad E(2\pi r L) = \frac{\rho \pi r^2 L}{\varepsilon_o}$$

Thus

$$E = \frac{\rho r}{2\varepsilon_o} \qquad \text{radially outward from the cylinder axis}$$

Notice that the electric field will increase as ρ increases, and also the electric field is proportional to r for $r<R$. For thr region outside the cylinder ($r>R$), the electric field will decrese as r increases.

Example 4.8

Two large non-conducting sheets of +ve charge face each other as shown in figure 4.20. What is E at points (i) to the left of the sheets (ii) between them and (iii) to the right of the sheets?

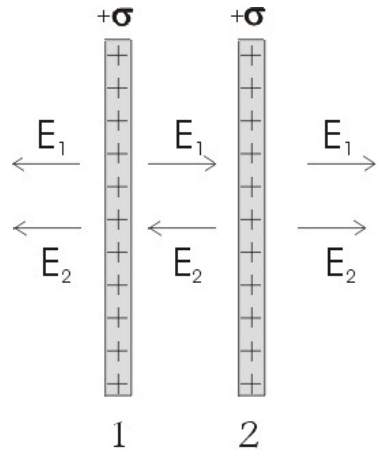


Figure 4.20

Solution

We know previously that for each sheet, the magnitude of the field at any point is

$$E = \frac{\sigma}{2\varepsilon_o}$$

(a) At point to the left of the two parallel sheets

$$E = -E_1 + (-E_2) = -2E$$

$$\therefore E = -\frac{\sigma}{\varepsilon_o}$$

(b) At point between the two sheets

$$E = E_1 + (-E_2) = \text{zero}$$

(c) At point to the right of the two parallel sheets

$$E = E_1 + E_2 = 2E$$

$$\therefore E = \frac{\sigma}{\varepsilon_o}$$

4.9 Problems

4.1) An electric field of intensity 3.5×103N/C is applied the x-axis. Calculate the electric flux through a rectangular plane 0.35m wide and 0.70m long if (a) the plane is parallel to the yz plane, (b) the plane is parallel to the xy plane, and (c) the plane contains the y axis and its normal makes an angle of 40^o with the x axis.

4.2) A point charge of +5μC is located at the center of a sphere with a radius of 12cm. What is the electric flux through the surface of this sphere?

4.3) (a) Two charges of 8μC and -5μC are inside a cube of sides 0.45m. What is the total electric flux through the cube? (b) Repeat (a) if the same two charges are inside a spherical shell of radius 0. 45 m.

4.4) The electric field everywhere on the surface of a hollow sphere of radius 0.75m is measured to be equal to 8.90×10^2N/C and points radially toward the center of the sphere. (a) What is the net charge within the surface? (b) What can you conclude about charge inside the nature and distribution of the charge inside the sphere?

4.5) Four closed surfaces, S_1, through S_4, together with the charges $-2Q$, $+Q$, and -Q are sketched in figure 4.21. Find the electric flux through each surface.

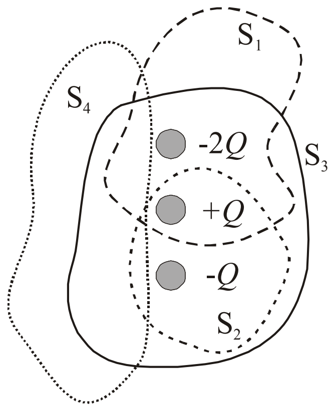


Figure 4.21

4.6) A conducting spherical shell of radius 15cm carries a net charge of -6.4μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

4.7) A long, straight metal rod has a radius of 5cm and a charge per unit length of 30nC/m. Find the electric field at the following distances from the axis of the rod: (a) 3cm, (b) 10cm, (c) 100cm.

4.8) A square plate of copper of sides 50cm is placed in an extended electric field of 8×10^4N/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

4.9) A solid copper sphere 15cm in radius has a total charge of 40nC. Find the electric field at the following distances measured from the center of the sphere: (a) 12cm, (b) 17cm, (c) 75cm. (d) How would your answers change if the sphere were hollow?

4.10) A solid conducting sphere of radius 2cm has a positive charge of +8μC. A conducting spherical shell d inner radius 4cm and outer radius 5cm is concentric with the solid sphere and has a net charge of -4μC. (a) Find the electric field at the following distances from the center of this charge configuration: (a) r=1cm, (b) r=3cm, (c) r=4.5cm, and (d) r=7cm.

4.11) A non-conducting sphere of radius a is placed at the center of a spherical conducting shell of inner radius b and outer radius c, A charge $+Q$ is distributed uniformly through the inner sphere (charge density ρC/m^3) as shown in figure 4.22. The outer shell carries $-Q$. Find $E(r)$ (i) within the sphere ($r<a$) (ii) between the sphere and the shell ($a<r<b$) (iii) inside the shell ($b<r<c$) and (iv) out side the shell and (v) What is the charge appear on the inner and outer surfaces of the shell?

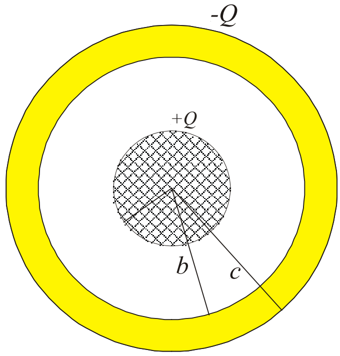


Figure 4.22

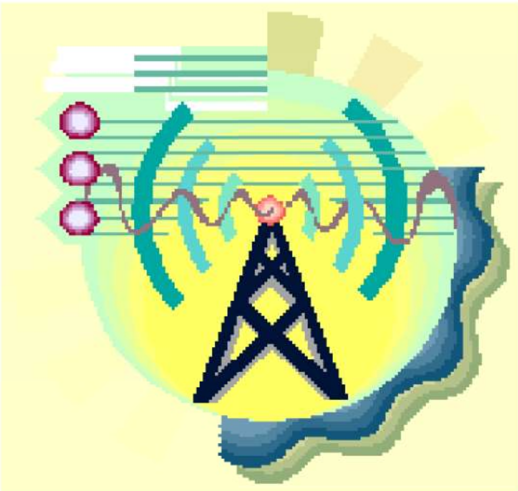
4.12) A solid sphere of radius 40cm has a total positive charge of 26μC uniformly distributed throughout its volume. Calculate the electric field intensity at the following distances from the center of the sphere: (a) 0 cm, (b) 10cm, (c) 40cm, (d) 60 cm.

4.13) An insulating sphere is 8cm in diameter, and carries a +5.7μC charge uniformly distributed throughout its interior volume. Calculate the charge enclosed by a concentric spherical surface with the following radii: (a) r=2cm and (b) r=6cm.

4.14) A long conducting cylinder (length l) carry a total charge $+q$ is surrounded by a conducting cylindrical shell of total charge $-2q$ as shown in figure 4.23. Use

Chapter 5

Electric Potential



الجهد الكهربي

Electric Potential

The Electric Potential

تعلمنا في الفصول السابقة كيف يمكن التعبير عن القوى الكهربية أو التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر باستخدام مفهوم المجال الكهربي. وكما نعلم أن المجال الكهربي هو كمية متجهة وقد استخدمنا لحسابه كلا من قانون كولوم وقانون جاوس. وقد سهل علينا قانون جاوس الكثير من التعقيدات الرياضية التي واجهتنا أثناء إيجاد المجال الكهربي لتوزيع متصل من الشحنة باستخدام قانون كولوم.

في هذه الفصل سوف نتعلم كيف يمكننا التعبير عن التأثير الكهربي في الفراغ المحيط بشحنة أو أكثر بواسطة كمية قياسية تسمى الجهد الكهربي The electric potential. وحيث أن الجهد الكهربي كمية قياسية وبالتالي فسيكون التعامل معه أسهل في التعبير عن التأثير الكهربي من المجال الكهربي.

في هذا الموضوع سندرس المواضيع التالية:-

(1) تعريف الجهد الكهربي.

(2) علاقة الجهد الكهربي بالمجال الكهربي.

(3) حساب الجهد الكهربي لشحنة في الفراغ.

(4) حساب المجال الكهربي من الجهد الكهربي.

(5) أمثلة ومسائل محلولة.

قبل أن نبدأ بتعريف الجهد الكهربي أو بمعنى أصح فرق الجهد الكهربي بين نقطتين في مجال شحنة في الفراغ سوف نضرب بعض الأمثلة التوضيحية.

مثال توضيحي (1)

عند رفع جسم كتلته m إلى ارتفاع h فوق سطح الأرض فإننا نقول أن شغلا خارجيا (موجبا) تم بذله لتحريك الجسم ضد عجلة الجاذبية الأرضية، وهذا الشغل سوف يتحول إلى طاقة وضع مختزنة في المجموعة المكونة من الجسم m والأرض. وطاقة الوضع هذه تزداد بازدياد المسافة h لأنه بالطبع سيزداد الشغل المبذول. إذا زال تأثير الشغل المبذول على الجسم m فإنه سيتحرك من المناطق ذات طاقة الوضع المرتفعة إلى المناطق ذات طاقة الوضع المنخفضة حتى يصبح فرق طاقة الوضع مساوياً للصفر.

مثال توضيحي (2)

نفرض إناء على شكل حرف U به ماء كما في شكل 5.1 . تكون طاقة الوضع لجزئ الماء عند النقطة B أكبر من طاقة الوضع عند النقطة A ولذلك إذا فتح الصنبور S فإن الماء سوف يتدفق في اتجاه النقطة A إلى أن يصبح الفرق في طاقتي الوضع بين النقطتين A&B مساويا للصفر.

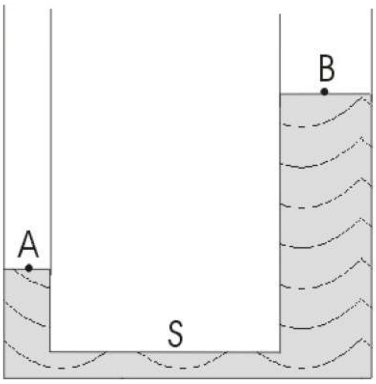


Figure 5.1

مثال توضيحي (3)

هناك حالة مشابهة تماما للحالتين السابقتين في الكهربية، حيث نفترض أن النقطتين A&B موجودتان في مجال كهربي ناتج من شحنة موجبة Q على سبيل المثال كما في شكل 5.2 . إذا كانت هناك شحنة اختبار q_o (مناظرة للجسم m في مجال عجلة الجاذبية الأرضية وكذلك لجزئ الماء عند النقطة B في المثال السابق) موجودة بالقرب من الشحنة Q فإن الشحنة q_o سوف تتحرك من نقطة قريبة من الشحنة إلى نقطة أكثر بعداً أي من B إلى A وفيزيائيا نقول أن الشحنة q_o تحركت من مناطق ذات جهد كهربي مرتفع إلى مناطق ذات جهد كهربي منخفض.

ولذلك يكون تعريف فرق الجهد الكهربي بين نقطتين A&B واقعتين في مجال كهربي شدته E بحساب الشغل المبذول بواسطة قوة خارجية (F_{ex}) ضد القوى الكهربية (qE) لتحريك شحنة اختبار q_o من A إلى B بحيث تكون دائما في حالة اتزان (أي التحريك بدون عجلة).

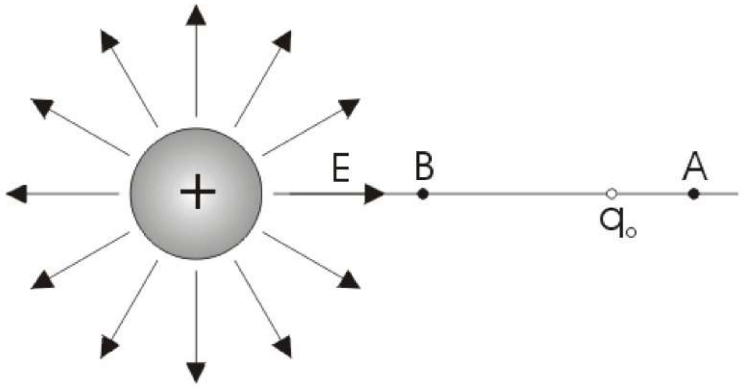


Figure 5.2

إذا كانت هنالك بطارية فرق الجهد بين قطبيها 1.5volt فهذا يعنى إنها إذا ما وصلت في دائرة كهربية، فإن الشحنات الموجبة ستتحرك من الجهد المرتفع إلى الجهد المنخفض. كما حدث في حالة فتح الصنبور في الأنبوبة U وستستمر حركة الشحنات حتى يصبح فرق الجهد بين قطبي البطارية مساوياً للصفر.

5.1 Definition of electric potential difference

We define the potential difference between two points A and B as the work done by an external agent in moving a test charge q_o from A to B *i.e.*

$$V_B - V_A = W_{AB} / q_o \qquad (5.1)$$

The unit of the potential difference is (***Joule/Coulomb***) which is known as ***Volt (V)***

Notice

Since the work may be (a) positive *i.e* $V_B > V_A$

(b) negative *i.e* $V_B < V_A$

(c) zero *i.e* $V_B = V_A$

You should remember that the work equals

$$W = \vec{F}_{ex}.\vec{l} = F_{ex}\cos\theta\, l$$

- If $0 < \theta < 90 \Rightarrow \cos\theta$ is +ve and therefore the W is +ve
- If $90 < \theta < 180 \Rightarrow \cos\theta$ is -ve and therefore W is -ve
- If $\theta = 90$ between F_{ex} and l $\Rightarrow$ therefore W is zero

The potential difference is independent on the path between A and B. Since the work (W_{AB}) done to move a test charge q_o from A to B is independent on the path, otherwise the work is not a scalar quantity. (see example 5.2)

5.2 The Equipotential surfaces

As the electric field can be represented graphically by lines of force, the potential distribution in an electric field may be represented graphically by equipotential surfaces.

The equipotential surface is a surface such that the potential has the same value at all points on the surface. *i.e.* $V_B - V_A =$ zero for any two points on one surface.

The work is required to move a test charge between any two points on an equipotential surface is zero. (*Explain why*?)

In all cases the equipotential surfaces are at right angles to the lines of force and thus to E. (*Explain why*?)

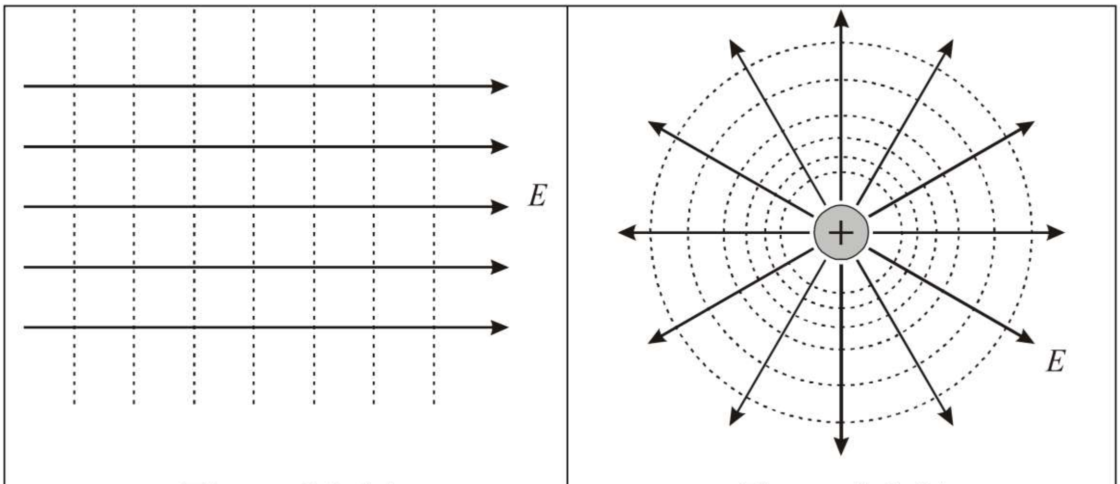


Figure 5.3 (a) ***Figure 5.3 (b)***

Figure 5.3 shows the equipotential surfaces (dashed lines) and the electric field lines (bold lines), (a) for uniform electric field and (b) for electric field due to a positive charge.

5.3 Electric Potential and Electric Field

Simple Case (Uniform electric field):

The potential difference between two points A and B in a Uniform electric field E can be found as follow,

Assume that a positive test charge q_o is moved by an external agent from A to B in uniform electric field as shown in figure 5.4.

The test charge q_o is affected by electric force of q_oE in the downward direction. To move the charge from A to B an external force F of the same magnitude to the electric force but in the opposite direction. The work W done by the external agent is:

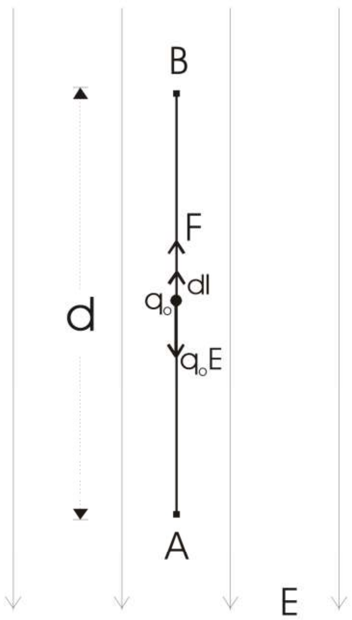


Figure 5.4

$$W_{AB} = Fd = q_oEd \qquad (5.2)$$

The potential difference V_B-V_A is

$$V_B - V_A = \frac{W_{AB}}{q_o} = Ed \qquad (5.3)$$

This equation shows the relation between the potential difference and the electric field for a special case (uniform electric field). Note that E has a new unit (V/m). hence,

$$\frac{Volt}{Meter} = \frac{Newton}{Coulomb}$$

The relation in general case (not uniform electric field):

If the test charge q_o is moved along a curved path from A to B as shown in figure 5.5. The electric field exerts a force q_oE on the charge. To keep the charge moving without accelerating, an external agent must apply a force F equal to $-q_oE$.

If the test charge moves distance dl along the path from A to B, the work done is $F.dl$. The total work is given by,

$$W_{AB} = \int_A^B \vec{F}.d\vec{l} = -q_o \int_A^B \vec{E}.d\vec{l} \quad (5.4)$$

The potential difference V_B-V_A is,

$$V_B - V_A = \frac{W_{AB}}{q_o} = -\int_A^B \vec{E}.d\vec{l} \quad (5.5)$$

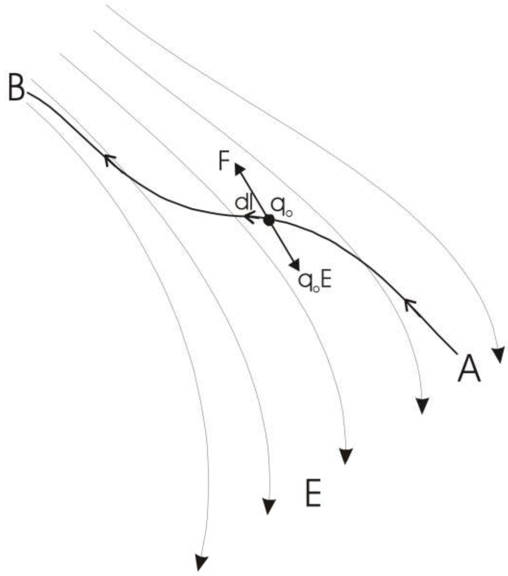


Figure 5.5

لاحظ هنا أن حدود التكامل من A إلى B هى التي تحدد المسار ومنه اتجاه متجه الإزاحة dl وتكون الزاوية θ هي الزاوية المحصورة بين منتجه الإزاحة ومتجه المجال الكهربي.

If the point A is taken to infinity then V_A=0 the potential V at point B is,

$$V_B = -\int_\infty^B \vec{E}.d\vec{l} \quad (5.6)$$

This equation gives the general relation between the potential and the electric field.

Example 5.1

Derive the potential difference between points *A* and *B* in uniform electric field using the general case.

Solution

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = -\int_A^B E\cos 180^o \, dl = \int_A^B Edl \qquad (5.7)$$

E is uniform (constant) and the integration over the path A to B is d, therefore

$$V_B - V_A = E\int_A^B dl = Ed \qquad (5.8)$$

Example 5.2

In figure 5.6 the test charge moved from *A* to *B* along the path shown. Calculate the potential difference between *A* and *B*.

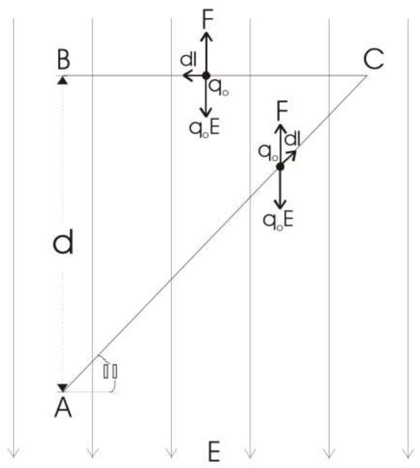


Figure 5.6

Solution

V_B-V_A=(V_B-V_C)+(V_C-V_A)

For the path *AC* the angle θ is 135°,

$$V_C - V_A = -\int_A^C \vec{E}.d\vec{l} = -\int_A^C E\cos 135^o\, dl = \frac{E}{\sqrt{2}}\int_A^C dl$$

The length of the line *AC* is $\sqrt{2}\, d$

$$V_C - V_A = \frac{E}{\sqrt{2}}(\sqrt{2}d) = Ed$$

For the path *CB* the work is zero and *E* is perpendicular to the path therefore, V_C-V_A = 0

$$V_B - V_A = V_C - V_A = Ed$$

The Electron Volt Unit

A widely used unit of energy in atomic physics is the electron volt (eV). **ELECTRON VOLT,** unit of energy, used by physicists to express the energy of ions and subatomic particles that have been accelerated in particle accelerators. One electron volt is equal to the amount of energy gained by an electron traveling through an electrical potential difference of 1 V; this is equivalent to 1.60207 x 10^{-19}J. Electron volts are commonly expressed as million electron volts (MeV) and billion electron volts (BeV or GeV).

5.4 Potential difference due to a point charge

Assume two points A and B near to a positive charge q as shown in figure 5.7. To calculate the potential difference V_B-V_A we assume a test charge q_o is moved without acceleration from A to B.

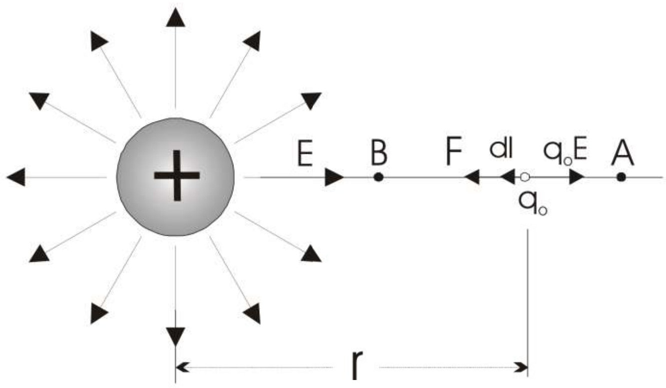


Figure 5.7

In the figure above the electric field E is directed to the right and dl to the left.

$$\vec{E}.d\vec{l} = E\cos 180^o\, dl = -Edl \qquad (5.10)$$

However when we move a distance dl to the left, we are moving in a direction of decreasing r. Thus

$$d\vec{l} = -d\vec{r} \qquad (5.11)$$

Therefore

$$-Edl = Edr \qquad (5.12)$$

$$\therefore V_B - V_A = -\int_A^B \vec{E}.d\vec{l} = \int_{r_A}^{r_B} \vec{E}.d\vec{r} \qquad (5.13)$$

Substitute for E

$$\because E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (5.14)$$

We get

$$\therefore V_B - V_A = -\frac{q}{4\pi\varepsilon_o}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \qquad (5.15)$$

لاحظ هنا أن هذا القانون يستخدم لإيجاد فرق الجهد الكهربي بين نقطتين في الفراغ المحيط بشحنة q.

5.5 The potential due to a point charge

If we choose *A* at infinity then V_A=0 (*i.e.* $r_A \Rightarrow \infty$) this lead to the potential at distance r from a charge *q* is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \qquad (5.16)$$

This equation shows that the equipotential surfaces for a charge are spheres concentric with the charge as shown in figure 5.8.

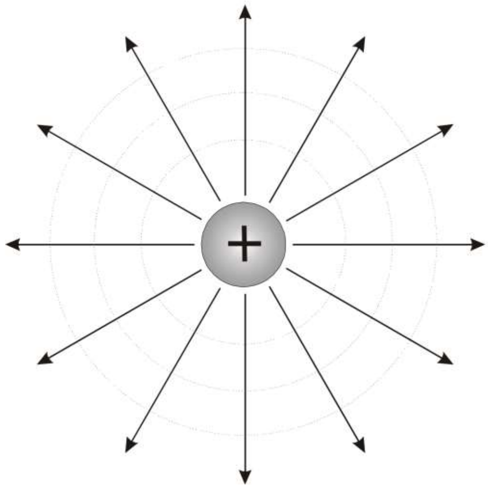


Figure 5.8

لاحظ أن المجال الكهربي لشحنة يتناسب عكسيا مع مربع المسافة، بينما الجهد الكهربي يتناسب عكسيا مع المسافة.

5.6 The potential due to a point charge

يمكن باستخدام هذا القانون إيجاد الجهد الكهربي لنقطة تبعد عن شحنة أو أكثر عن طريق الجمع الجبري للجهد الكهربي الناشئ عن كل شحنة على حده عند النقطة المراد إيجاد الجهد الكلى عندها أي

$$V = V_1 + V_2 + V_3 + ... + V_n \qquad (5.17)$$

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o} \sum_n \frac{q_n}{r_n} \qquad (5.18)$$

عند التعويض عن قيمة الشحنة q تأخذ الإشارة في الحسبان، لأنك تجمع جمعاً جبرياً هنا وليس جمعاً اتجاهياً كما كنا نفعل في المجال الكهربي حيث تحدد الإشارة الاتجاه على الرسم.

Example 5.3

What must the magnitude of an isolated positive charge be for the electric potential at 10 cm from the charge to be +100V?

Solution

$$V = \frac{1}{4\pi\varepsilon_o} \frac{q}{r}$$

$$\therefore q = V 4\pi\varepsilon_o r^2 = 100 \times 4\pi \times 8.9 \times 10^{-12} \times 0.1 = 1.1 \times 10^{-9} C$$

Example 5.4

What is the potential at the center of the square shown in figure 5.9? Assume that q_1= +1 ×10^{-8}C, q_2= -2×10^{-8}C, q_3=+3×10^{-8}C, q_4=+2×10^{-8}C, and a=1m.

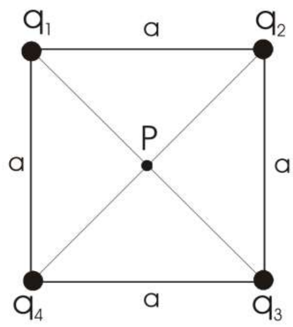


Figure 5.9

Solution

$$\therefore V = \sum_n V_n = \frac{1}{4\pi\varepsilon_o}\frac{q_1 + q_2 + q_3 + q_4}{r}$$

The distance r for each charge from P is 0.71m

$$\therefore V = \frac{9\times10^{9}(1-2+3+2)\times10^{-8}}{0.71} = 500V$$

Example 5.5

Calculate the electric potential due to an electric dipole as shown in figure 5.10.

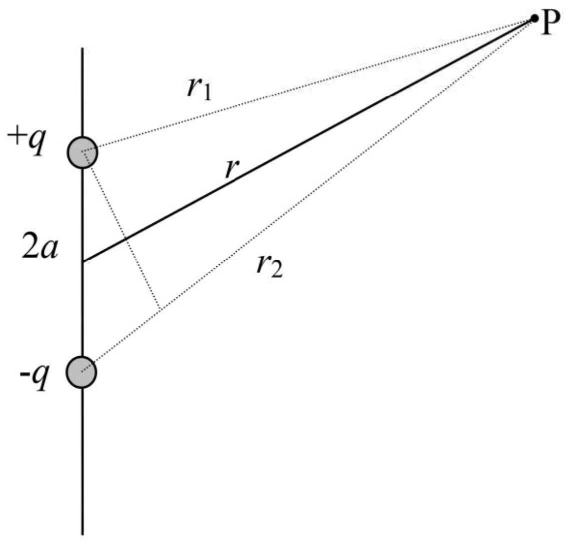


Figure 5.10

Solution

$V = \sum V_n = V_1 + V_2$

$$V = K\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = Kq\frac{r_2 - r_1}{r_2 r_1}$$

When $r >> 2a$,

$r_2 - r_1 \cong 2a\cos\theta$ and $r_1 r_2 \cong r^2$,

$$V = Kq\frac{2a\cos\theta}{r^2} = K\frac{p\cos\theta}{r^2} \qquad (5.19)$$

where p is the dipole momentum

Note that $V = 0$ when $\theta=90^{o}$ but V has the maximum positive value when $\theta=0^{o}$ and V has the maximum negative value when $\theta=180^{o}$.

5.7 Electric Potential Energy

The definition of the ***electric potential energy*** of a system of charges is the work required to bring them from infinity to that configuration.

To workout the electric potential energy for a system of charges, assume a charge q_2 at infinity and at rest as shown in figure 5.11. If q_2 is moved from infinity to a distance r from another charge q_1, then the work required is given by

$$W=Vq_2$$

$$\because V=\frac{1}{4\pi\varepsilon_o}\frac{q_1}{r}$$

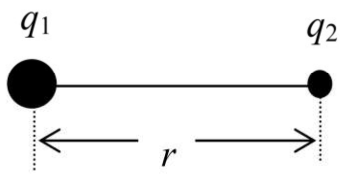


Figure 5.11

Substitute for V in the equation of work

$$U=W=\frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r_{12}} \qquad (5.20)$$

$$U=\frac{q_1q_2}{4\pi\varepsilon_o r} \qquad (5.21)$$

To calculate the potential energy for systems containing more than two charges we compute the potential energy for every pair of charges separately and to add the results algebraically.

$$U=\sum\frac{q_iq_j}{4\pi\varepsilon_o r_{ij}} \qquad (5.22)$$

القانون الأول يطبق في حالة شحنتين فقط، ولكن إذا كانت المجموعة المراد إيجاد طاقة الوضع الكهربي لها أكثر من شحنتين نستخدم القانون الثاني حيث نوجد الطاقة المختزنة بين كل شحنتين على حده ثم نجمع جمعا جبريا، أي نعوض عن قيمة الشحنة ونأخذ الإشارة بالحسبان في كل مرة.

If the total electric potential energy of a system of charges is positive this correspond to a repulsive electric forces, but if the total electric potential energy is negative this correspond to attractive electric forces. (*explain why*?)

Example 5.6

Three charges are held fixed as shown in figure 5.12. What is the potential energy? Assume that q=1 $\times 10^{-7}$C and a=10cm.

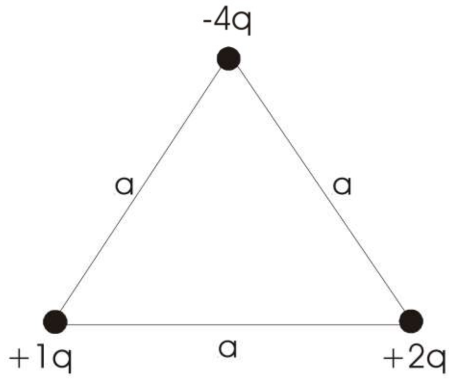


Figure 5.12

Solution

$$U=U_{12}+U_{13}+U_{23}$$

$$U=\frac{1}{4\pi\varepsilon_o}\left[\frac{(+q)(-q)}{a}+\frac{(+q)(+2q)}{a}+\frac{(-4q)(+2q)}{a}\right]$$

$$U=-\frac{10}{4\pi\varepsilon_o}\frac{q^2}{a}$$

$$\therefore U=-\frac{9\times10^9(10)(1\times10^{-7})^2}{0.1}=-9\times10^{-3}J$$

نلاحظ أن قيمة الطاقة الكلية سالبة، وهذا يعني أن الشغل المبذول للحفاظ على ثبات الشحنات سابقة الذكر سالب أيضاً. نستنتج من ذلك أن القوة المتبادلة بين الشحنات هي قوة تجاذب، أما في حالة أن تكون الطاقة الكلية موجبة فإن هذا يعني أن القوة المتبادلة بين الشحنات هي قوة تنافر.

5.8 Calculation of *E* from *V*

As we have learned that both the electric field and the electric potential can be used to evaluate the electric effects. Also we have showed how to calculate the electric potential from the electric field now we determine the electric field from the electric potential by the following relation.

$$\vec{E} = -\frac{dV}{dl} \qquad (5.23)$$

New unit for the electric field is *volt/meter* (v/m)

لاحظ أن العلاقة الرياضية بين المجال الكهربي والجهد الكهربي هي علاقة تفاضل وتكامل وبالتالي إذا علمنا الجهد الكهربي يمكن بإجراء عملية التفاضل إيجاد المجال الكهربي. وتذكر أن خطوط المجال الكهربي عمودية على أسطح متساوية الجهد equipotential surfaces.

Example 5.7

Calculate the electric field for a point charge *q*, using the equation
$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Solution

$$E = -\frac{dV}{dl} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r}\right)$$

$$E = -\frac{q}{4\pi\varepsilon_o}\frac{d}{dr}\left(\frac{1}{r}\right) = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

5.9 Solution of some selected problems

في هذا الجزء سنعرض حلولاً لبعض المسائل التي تغطى موضوع الجهد الكهربي والمجال الكهربي

Example 5.8

Two charges of 2μC and -6μC are located at positions (0,0) m and (0,3) m, respectively as shown in figure 5.13. (i) Find the total electric potential due to these charges at point (4,0) m.

(ii) How much work is required to bring a 3μC charge from ∞ to the point *P*?

(iii) What is the potential energy for the three charges?

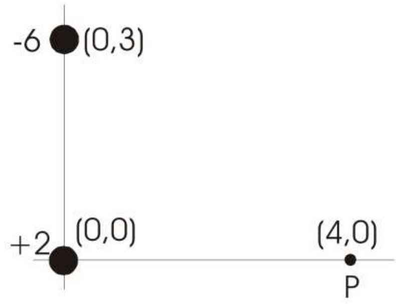


Figure 5.13

Solution

$$V_p = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$V = 9\times10^9\left[\frac{2\times10^{-6}}{4} - \frac{6\times10^{-6}}{5}\right] = -6.3\times10^3 \text{ volt}$$

(ii) the work required is given by

$$W = q_3\, V_p = 3\times10^{-6}\times -6.3\times10^3 = -18.9\times10^{-3}\text{ J}$$

The -ve sign means that work is done by the charge for the movement from ∞ to *P*.

(iii) The potential energy is given by

$$U = U_{12} + U_{13} + U_{23}$$

$$U = k\left[\frac{(2\times10^{-6})(-6\times10^{-6})}{3} + \frac{(2\times10^{-6})(3\times10^{-6})}{4} + \frac{(-6\times10^{-6})(3\times10^{-6})}{5}\right]$$

$$\therefore U = -5.5\times10^{-2}\ \textit{Joule}$$

Example 5.9

A particle having a charge q=3×10^{-9}C moves from point a to point b along a straight line, a total distance d=0.5m. The electric field is uniform along this line, in the direction from a to b, with magnitude E=200N/C. Determine the force on q, the work done on it by the electric field, and the potential difference V_a-V_b.

Solution

The force is in the same direction as the electric field since the charge is positive; the magnitude of the force is given by

$$F = qE = 3\times10^{-9} \times 200 = 600\times10^{-9}\text{N}$$

The work done by this force is

$$W = Fd = 600\times10^{-9} \times 0.5 = 300\times10^{-9}\text{J}$$

The potential difference is the work per unit charge, which is

$$V_a - V_b = W/q = 100\text{V}$$

Or

$$V_a - V_b = Ed = 200 \times 0.5 = 100\text{V}$$

Example 5.10

Point charge of +12×10^{-9}C and -12×10^{-9}C are placed 10cm part as shown in figure 5.14. Compute the potential at point a, b, and c.

Compute the potential energy of a point charge +4×10^{-9}C if it placed at points a, b, and c.

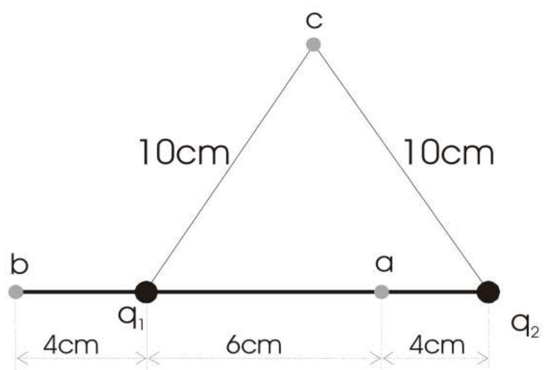


Figure 5.14

Solution

We need to use the following equation at each point to calculate the potential,

$$V=\sum_n V_n=\frac{1}{4\pi\varepsilon_o}\sum\frac{q_i}{r_i}$$

At point *a*

$$V_a=9\times10^9\left(\frac{12\times10^{-9}}{0.06}+\frac{-12\times10^{-9}}{0.04}\right)=-900V$$

At point *b*

$$V_b=9\times10^9\left(\frac{12\times10^{-9}}{0.04}+\frac{-12\times10^{-9}}{0.14}\right)=-1930V$$

At point *c*

$$V_c=9\times10^9\left(\frac{12\times10^{-9}}{0.1}+\frac{-12\times10^{-9}}{0.14}\right)=0V$$

We need to use the following equation at each point to calculate the potential energy,

$$U=qV$$

At point *a*

$$U_a = qV_a = 4\times10^{-9}\times(-900) = -36\times10^{-7}\text{J}$$

At point *b*

$$U_b = qV_b = 4\times10^{-9}\times1930 = +77\times10^{-7}\text{J}$$

At point *c*

$$U_c = qV_c = 4\times10^{-9}\times0 = 0$$

Example 5.11

A charge q is distributed throughout a nonconducting spherical volume of radius R. (a) Show that the potential at a distance r from the center where $r < R$, is given by

$$V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

Solution

لايجاد الجهد داخل الكرة غير الموصلة عند نقطة A مثلا فإننا سوف نحسب فرق الجهد بين موضع في مالانهاية والنقطة A.

$$V_B - V_\infty = -\int \vec{E}.d\vec{l}$$

وحيث أن للمجال قيمتين مختلفتين خارج الكرة وداخلها كما نعلم من مسألة سابقة من مسائل قانون جاوس.

$$E_{out} = \frac{q}{4\pi\varepsilon_o r^2} \qquad E_{in} = \frac{qr}{4\pi\varepsilon_o R^3}$$

$$V_A - V_\infty = (V_A - V_B) + (V_B - V_\infty)$$

$$V_A - V_\infty = -\int \vec{E}_{in}.d\vec{l} - \int \vec{E}_{out}.d\vec{l}$$

نلاحظ أن الزاوية بين dl & E هي 180^o أي أن $\cos 180 = -1$ ولكن أيضا $dl = -dr$

$$V_A - V_\infty = -\int \frac{qr}{4\pi\varepsilon_o R^3}\, dr - \int \frac{q}{4\pi\varepsilon_o r^2}\, dr$$

$$= -\frac{q}{4\pi\varepsilon_o R^3}\left[\frac{r^2}{2}\right] + \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}\right] = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$

وهذا هو الجهد عند النقطة A وهو المطلوب إثباته

إذا كانت A على سطح الكرة فإن الجهد في هذه الحالة $V = \dfrac{q}{4\pi\varepsilon_o R}$

أي كالجهد الكهربي لشحنة في الفراغ.

Example 5.12

For the charge configuration shown in figure 5.15, Show that $V(r)$ for the points on the vertical axis, assuming $r >> a$, is given by

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{2aq}{r^2}\right]$$

Solution

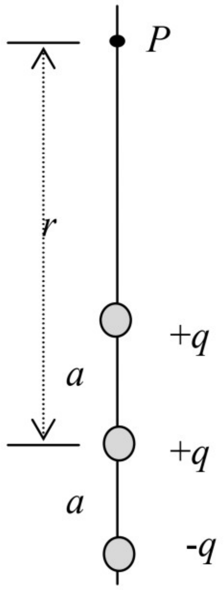


Figure 5.15

$V_p = V_1 + V_2 + V_3$

$$V = \frac{q}{4\pi\varepsilon_o(r-a)} + \frac{q}{4\pi\varepsilon_o r} - \frac{q}{4\pi\varepsilon_o(r+a)}$$

$$\frac{q(r+a)-q(r-a)}{4\pi\varepsilon_o(r^2-a^2)} + \frac{q}{4\pi\varepsilon_o r}$$

$$\frac{2aq}{4\pi\varepsilon_o r^2(1-a^2/r^2)} + \frac{q}{4\pi\varepsilon_o r}$$

when $r>>a$ then $a^2/r^2 <<1$

$$V = \frac{2aq}{4\pi\varepsilon_o r^2}(1-a^2/r^2)^{-1} + \frac{q}{4\pi\varepsilon_o r}$$

يمكن فك القوسين بنظرية ذات الحدين والاحتفاظ بأول حدين فقط كتقريب جيد

$(1+x)^n = 1 + nx$ when $x<<1$

$$V = \frac{2aq}{4\pi\varepsilon_o r^2}(1+a^2/r^2) + \frac{q}{4\pi\varepsilon_o r}$$

ويمكن إهمال a^2/r^2 بالنسبة لـ 1 .

$$\therefore V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{r} + \frac{2aq}{r^2}\right]$$

Example 5.13

Derive an expression for the work required to put the four charges together as indicated in figure 5.16.

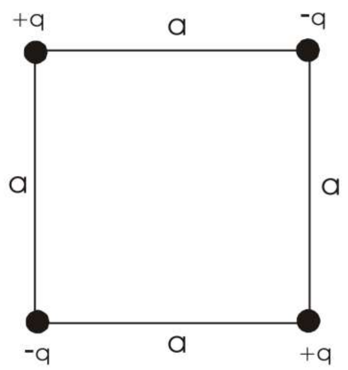


Figure 5.16

Solution

The work required to put these charges together is equal to the total electric potential energy.

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-4q^2}{a} + \frac{2q^2}{\sqrt{2}a}\right]$$

$$U = \frac{1}{4\pi\varepsilon_o}\left[\frac{-\sqrt{2}4q^2 + 2q^2}{\sqrt{2}a}\right] = \frac{-0.2q^2}{\varepsilon_o a}$$

The minus sign indicates that there is attractive force between the charges

---

In Example 5.13 assume that if all the charges are positive, prove that the work required to put the four charges together is

$$U = \frac{1}{4\pi\varepsilon_o}\frac{5.41q^2}{\varepsilon_o a}$$

Example 5.14

In the rectangle shown in figure 5.17, q_1 = -5x10^{-6}C and q_2 = 2x10^{-6}C calculate the work required to move a charge q_3 = 3x10^{-6}C from *B* to *A* along the diagonal of the rectangle.

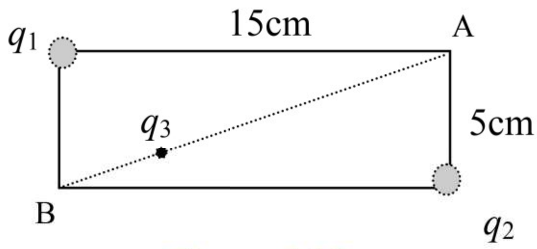


Figure 5.17

Solution

from the equation $\quad V_B - V_A = W_{AB} / q_o$

$V_A = V_1 + V_2 \quad \& \quad V_B = V_1 + V_2$

$$V_A = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.15} + \frac{2\times10^{-6}}{0.05}\right] = 6 \times 10^4 \text{ V}$$

$$V_B = \frac{q}{4\pi\varepsilon_o}\left[\frac{-5\times10^{-6}}{0.05} + \frac{2\times10^{-6}}{0.15}\right] = -7.8 \times 10^4 \text{ V}$$

$$W_{BA} = (V_A - V_B)\, q_3$$

$$=(6 \times 10^4 + 7.8 \times 10^4)\ 3 \times 10^{-6} = 0.414 \text{ Joule}$$

Example 5.15

Two large parallel conducting plates are 10 cm a part and carry equal but opposite charges on their facing surfaces as shown in figure 5.18. An electron placed midway between the two plates experiences a force of 1.6×10^{-15} N.

What is the potential difference between the plates?

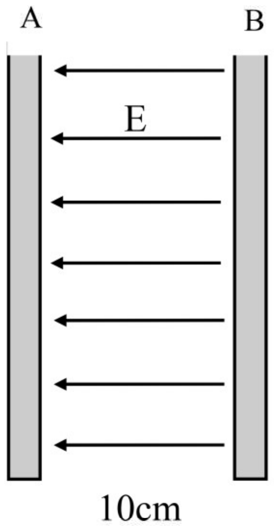


Figure 5.18

Solution

$$V_B - V_A = Ed$$

يمكن حساب المجال الكهربي عن طريق القوى الكهربية المؤثرة على الإلكترون

$$F = eE \Rightarrow E = F/e$$

$$V_B - V_A = 10000 \times 0.1 = 1000 \text{ volt}$$

5.10 Problems

5.1) What potential difference is needed to stop an electron with an initial speed of 4.2×10^5m/s?

5.2) An ion accelerated through a potential difference of 115V experiences an increase in potential energy of 7.37×10^{-17}J. Calculate the charge on the ion.

5.3) How much energy is gained by a charge of 75 μC moving through a potential difference of 90V?

5.4) An infinite charged sheet has a surface charge density σ of 1.0×10^{-7} C/m^2. How far apart are the equipotential surfaces whose potentials differ by 5.0 V?

5.5) At what distance from a point charge of 8μC would the potential equal 3.6×10^4V?

5.6) At a distance r away from a point charge q, the electrical potential is V=400V and the magnitude of the electric field is E=150N/C. Determine the value of q and r.

5.7) Calculate the value of the electric potential at point P due to the charge configuration shown in Figure 5.19. Use the values q_1=5μC, q_2=-10μC, a=0.4m, and b=0.5m.

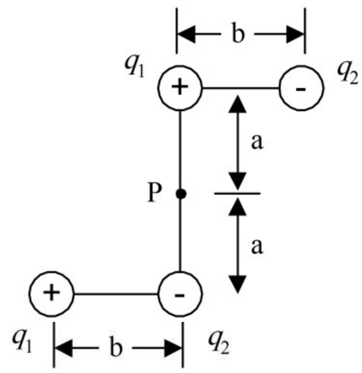


Figure 5.19

5.8) Two point charges are located as shown in Figure 5.20, where q_1=+4μC, q_2=-2μC, a=0.30m, and b =0.90m. Calculate the value of the electrical potential at points P_1, and P_2. Which point is at the higher potential?

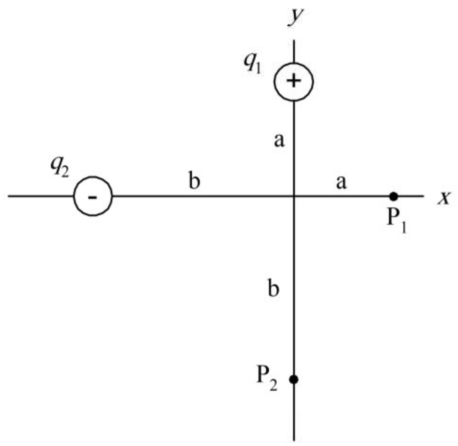


Figure 5.20

5.9) Consider a point charge with $q=1.5\times10^{-6}$C. What is the radius of an equipotential surface having a potential of 30V?

5.10) Two large parallel conducting plates are 10cm apart and carry equal and opposite charges on their facing surfaces. An electron placed midway between the two plates experiences a force of 1.6×10^{15}N. What is the potential difference between the plates?

5.11) A point charge has $q=1.0\times10^{-6}$C. Consider point A which is 2m distance and point B which is 1m distance as shown in the figure 5.21(a). (a) What is the potential difference V_A-V_B? (b) Repeat if points A and B are located differently as shown in figure 5.21(b).

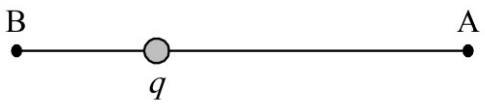


Figure 5.21(a)

Figure 5.21(b)

5.12) In figure 5.22 prove that the work required to put four charges together on the corner of a square n by (w=-$0.21q^2/\varepsilon_o$ a).

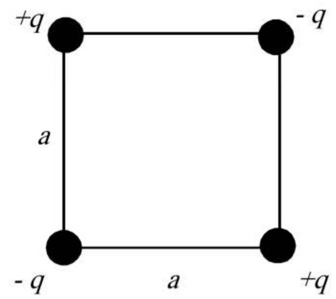


Figure 5.22

5.13) Two charges $q=+2\times10^{-6}$C are fixed in space a distance d=2cm) apart, as shown in figure 5.23 (a) What is the electric potential at point C? (b) You bring a third charge $q=2.0\times10^{-6}$C very slowly from infinity to C. How much work must you do? (c) What is the potential energy U of the configuration when the third charge is in place?

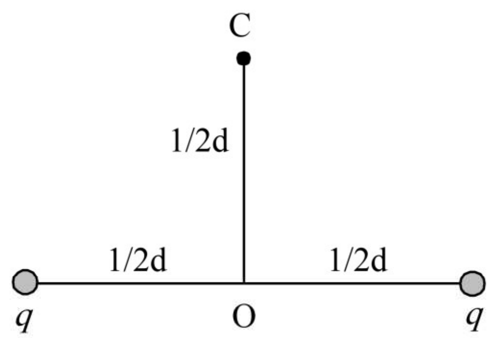


Figure 5.23

5.14) Four equal point charges of charge q=+5μC are located at the corners of a 30cm by 40cm rectangle. Calculate the electric potential energy stored in this charge configuration.

5.15) Two point charges, Q_1=+5nC and Q_2=-3nC, are separated by 35cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

Multiple Choice Questions

Part 1
Principles of Electrostatic

Coulomb's Law
Electric Field
Gauss's Law
Electric Potential Difference

Attempt the following question after the completion of part 1

[1] Two small beads having positive charges 3 and 1 are fixed on the opposite ends of a horizontal insulating rod, extending from the origin to the point x=d. As in Figure 1, a third small, charged bead is free to slide on the rod. At what position is the third bead in equilibrium?

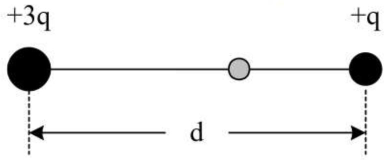


Figure 1

a. $x = 0.366d$
b. $x = 0.634d$
c. $x = 0.900d$
d. $x = 2.37d$

---

[2] Two identical conducting small spheres are placed with their centers 0.300m apart. One is given a charge of 12.0nC and the other one a charge of 18.0nC. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. After equilibrium has occurred, find the electrostatic force between the two.

a. (a) 2.16×10^{-5} N attraction; (b) 0 N repulsion
b. (a) 6.47×10^{-6} N repulsion; (b) 2.70×10^{-7} N attraction
c. (a) 2.16×10^{-5} N attraction; (b) 8.99×10^{-7} N repulsion
d. (a) 6.47×10^{-6} N attraction; (b) 2.25×10^{-5} N repulsion

---

[3] An electron is projected at an angle of 40.0° above the horizontal at a speed of 5.20×10^5 m/s in a region where the electric field is E = 3 50 j N/C. Neglect gravity and find (a) the time it takes the electron to return to its maximum height, (h) the maximum height it reaches and (c) its horizontal displacement when it reaches its maximum height.

a. (a) 1.09×10^{-8} s; (b) 0.909 mm; (c) 2.17 m
b. (a) 1.69×10^{-8} s; (b) 2.20 mm; (c) 4.40 m
c. (a) 1.09×10^{-8} s; (b) 4.34 mm; (c) 0.909 m
d. (a) 1.30×10^{-8} s; (b) 1.29 mm; (c) 2.17 m

---

[4] Two identical metal blocks resting on a frictionless horizontal surface are connected by a light metal spring for which the spring constant is *k* 175 N/m and the unscratched length is 0.350 m as in Figure 2a.

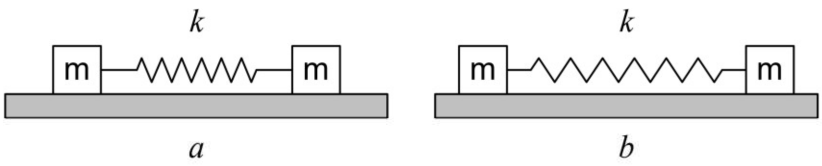


Figure 2

A charge Q is slowly placed on the system causing the spring to stretch to an equilibrium length of 0.460 m as in Figure 2b. Determine the value of Q, assuming that all the charge resides in the blocks and that the blocks can be treated as point charges.

a. 64.8 μC
b. 32.4 μC
c. 85.1 μC
d. 42.6 μC

---

[5] A small plastic ball 1.00 g in mass is suspended by a 24.0 cm long string in a uniform electric field as shown in Figure P23.52.

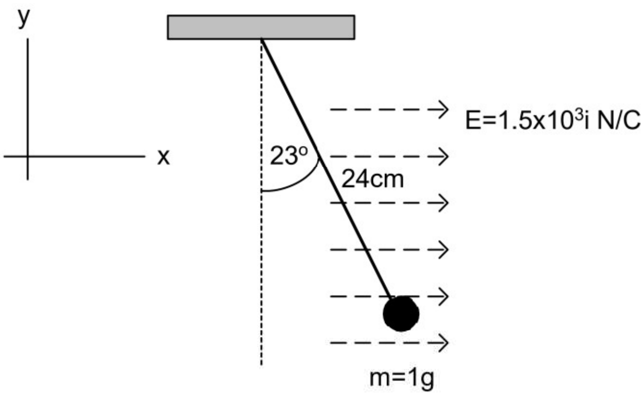


Figure 3

If the ball is in equilibrium when the string makes a 23.0° angle with the vertical, what is the net charge on the ball?

a. 36.1 μC
b. 15.4 μC
c. 6.5 3 μC
d. 2.77 μC

---

[6] An object having a net charge of 24.0 μC is placed in a uniform electric field of 6 10 N/C directed vertically. What is the mass of the object if it "floats" in the field?

a. 0.386 g
b. 0.669 g
c. 2.59 g
d. 1.49 g

---

[7] Four identical point charges (q = +14.0 μC) are located on the corners of a rectangle as shown in Figure 4.

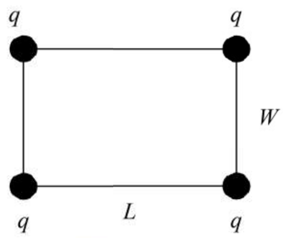


Figure 4

The dimensions of the rectangle are L = 55.0 cm and W= 13.0 cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges. (Call the lower left corner of the rectangle the origin.)

a. 106 mN @ 264°
b. 7.58 mN @ 13.3°
c. 7.58 mN @ 84.0°
d. 106 mN @ 193°

---

[8] An electron and proton are each placed at rest in an electric field of 720 N/C. Calculate the speed of each particle 44.0 ns after being released.

a. $v_e = 1.27 \times 10^6$ m/S, $v_p = 6.90 \times 10^3$ m/s
b. $v_e = 5.56 \times 10^6$ m/S, $v_p = 3.04 \times 10^3$ m/s
c. $v_{e.} = 1.27 \times 10^{14}$ m/S, $v_p = 6.90 \times 10^{10}$ m/s
d. $v_e = 3.04 \times 10^3$ m/S, $v_p = 5.56 \times 10^6$ m/s

---

[9] Three point charges are arranged as shown in Figure 5.

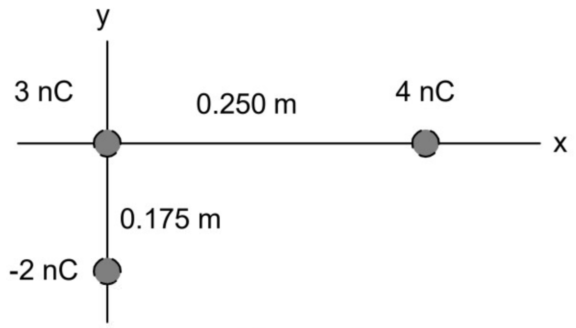


Figure 5

(a) Find the vector electric field that the 4.00 nC and -2.00 nC charges together create at the origin. (b) Find the vector force on the 3.00 nC charge.

a. (a) (0.144i - 0.103 j) kN/C; (b) (0.432i - 0.308j) μN
b. (a) (-0.575i - 0.587j) kN/C; (b) (-1.73i- 1.76j) μN
c. (a) (-0.144i - 0.103j) kN/C; (b) (-0.432i - 0.308j) μN
d. (a) (-0.575i + 0.587j) kN/C; (b) (-1.73i + 1.76j) μN

---

[10] Two 1.00 μC point charges are located on the x axis. One is at x = 0.60 m, and the other is at x = -0.60 m. (a) Determine the electric field on the y axis at x = 0.90 m. (b) Calculate the electric force on a -5.00 μC charge placed on the y axis at y = 0.90 m.

a. (a) $(8.52 \times 10^3 i + 1.28 \times 10^4 j)$N/C; (b) $(-4.62 \times 10^{-2} i - 6.39 \times 10^{-2} j)$N
b. (a) $8.52 \times 10^3 j$ N/C; (b) $-4.26 \times 10^{-2} j$ N
c. (a) $1.28 \times 10^4 j$ N/C; (b) $-6.39 \times 10^{-2} j$ N
d. (a) -7.68×10^3N/C; (b) $3.84 \times 10^{-2} j$ N

---

[11] A 14.0μC charge located at the origin of a cartesian coordinate system is surrounded by a nonconducting hollow sphere of radius 6.00 cm. A drill with a radius of 0.800 mm is aligned along the z-axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

a. 176 Nm^2/C
b.4.22 Nm^2/C
c. 0 Nm^2/C
d. 70.3 Nm^2/C

---

[12] An electric field of intensity 2.50 kN/C is applied along the x-axis. Calculate the electric flux through a rectangular plane 0.450 m wide and

0.800 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the *xy* plane; (c) the.plane contains the y-axis and its normal makes an angle of 30.0° with the x-axis.

a. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 779 Nm^2/C
b. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 779 Nm^2/C
c. (a) 0 Nm^2/C; (b) 900 Nm^2/C; (c) 450 Nm^2/C
d. (a) 900 Nm^2/C; (b) 0 Nm^2/C; (c) 450 Nm^2/C

---

[13] A conducting spherical shell of radius 13.0 cm carries a net charge of -7.40 μC uniformly distributed on its surface. Find the electric field at points (a) just outside the shell and (b) inside the shell.

a. (a) (-7.88 mN/C)r; (b) (-7.88 mN/C)r
b. (a) (7.88 mN/C)r; (b) (0 mN/C)r
c. (a) (-3.94 rnN/C)r; (b) (0 mN/C)r
d. (a) (3.94 mN/C)r; (b) (3.94 mN/C)r

---

[14] A point charge of 0.0562 μC is inside a pyramid. Determine the total electric flux through the surface of the pyramid.

a.1.27×10^3 Nm^2/C^2
b.6.35×10^3 Nm^2/C^2
c. 0 Nm^2/C^2
d. 3.18×10^4 Nm^2/C^2

---

[15] A large flat sheet of charge has a charge per unit area of 7.00 $\mu C/m^2$. Find the electric field intensity just above the surface of the sheet, measured from its midpoint.

a. 7.91×10^5 N/C up
b. 1.98×10^5 N/C up
c. 3.95×10^5 N/C up
d. 1.58×10^6 N/C up

---

[16] The electric field on the surface of an irregularly shaped conductor varies from 60.0 kN/C to 24.0 kN/C. Calculate the local surface charge

density at the point on the surface where the radius of curvature of the surface is (a) greatest and (b) smallest.

a. 0.531 $\mu C/m^2$; (b) 0.2l2, $\mu C/m^2$
b. l.06, $\mu C/m^2$; (b) 0.425 $\mu C/m^2$
c. 0.425, $\mu C/m^2$; (b) 1.06$\mu C/m^2$
d. 0.212 $\mu C/m^2$; (b) 0.531 $\mu C/m^2$

---

[17] A square plate of copper with 50.0 cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicular to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

a. (a) $\sigma = \pm$ 0.708 $\mu C/m^2$; (b) $Q =\pm$ 0.0885 μC
b. (a) $\sigma = \pm$ 1.42 $\mu C/m^2$; (b) $Q = \pm$ 0.354 μC
c. (a) $\sigma = \pm$ 0.708 $\mu C/m^2$; (b) $Q = \pm$ 0.177 μC
d. (a) $\sigma = \pm$ 1.42 $\mu C/m^2$; (b) $Q = \pm$ 0.177 μC

---

[18] The following charges are located inside a submarine: 5.00μC, -9.00μC, 27.0μC and -84.0μC. (a) Calculate the net electric flux through the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

a. (a) 1.41×10^7 Nm^2/C; (b) greater than
b. (a) -6.89×10^6 Nm^2/C; (b) less than
c. (a) -6.89×10^6 Nm^2/C; (b) equal to
d. (a) 1.41×10^7 Nm^2/C; (b) equal to

---

[19] A solid sphere of radius 40.0 cm has a total positive charge of 26.0μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field at 90.0 cm.

a. (2.89×10^5 N/C)r
b. (3.29×10^6 N/C)r
c. 0 N/C
d. (1.46×10^6 N/C)r

---

[20] A charge of 190 μC is at the center of a cube of side 85.0 cm long. (a) Find the total flux through each face of the cube. (b) Find the flux through the whole surface of the cube.

a. (a) 3.58×10^6 Nm^2/C; (b) 2.15×10^7 Nm^2/C
b. (a) 4.10×10^7 Nm^2/C; (b) 4.10×10^7 Nm^2/C
c. (a) 1.29×10^8 Nm^2/C; (b) 2.15×10^7 Nm^2/C
d. (a) 6.83×10^6 Nm^2/C; (b) 4.10×10^7 Nm^2/C

---

[21] A 30.0 cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is found to be 3.20 x 105 Nm2/C. What is the electric field strength?

a. 3.40×10^5 N/C
b. 4.53×10^6 N/C
c. 1.13×10^6N/C
d. 1.70×10^5 N/C

---

[22] Consider a thin spherical shell of radius 22.0 cm with a total charge of 34.0μC distributed uniformly on its surface. Find the magnitude of the electric field (a) 15.0 cm and (b) 30.0 cm from the center of the charge distribution.

a. (a) 6.32×10^6 N/C; (b) 3.40×10^6 N/C
b. (a) 0 N/C; (b) 6.32×10^6 N/C
c. (a) 1.36×10^7 N/C; (b) 3.40×10^6 N/C
d. (a) 0 N/C; (b) 3.40×10^6 N/C

---

[23] A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field 100.0 cm from the axis of the rod, where distances area measured perpendicular to the rod.

a. (1.08×10^4 N/C)r
b. (2.70×10^2 N/C)r
c. (5.39×10^2 N/C)r
d. (0 N/C)r

---

[24] A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of -4.00 μC. Find the electric field at r = 7.00 cm from the center of this charge configuration.

a. $(2.20 \times 10^7$ N/C)r
b. $(4.32 \times 10^7$ N/C)r
c. $(7.34 \times 10^6$ N/C)r
d. $(1.44 \times 10^7$ N/C)r

---

[25] The electric field everywhere on the surface of a thin spherical shell of radius 0.650 m is measured to be equal to 790 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What can you conclude about the nature and distribution of the charge inside the spherical shell?

a. (a) 3.71×10^{-8}C; (b) The charge is negative, its distribution is spherically symmetric.
b. (a) 3.71×10^{-8} C; (b) The charge is positive, its distribution is uncertain.
c. (a) 1.93×10^{-4} C; (b) The charge is positive, its distribution is spherically symmetric.
d. (a) 1.93×10^{-4} C; (b) The charge is negative, its distribution is uncertain.

---

[26] Four identical point charges (q = +16.0 μC) are located on the corners of a rectangle, as shown in Figure 6.

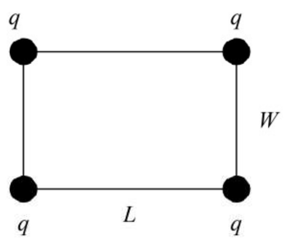


Figure 6

The dimensions of the rectangle are L 70.0 cm and W= 30.0 cm. Calculate the electric potential energy of the charge at the lower left corner due to the other three charges.

a. 14.9 J

b. 7.94 J
c. 14.0 J
d. 34.2 J

[27] The three charges in Figure 7 are at the vertices of an isosceles triangle.

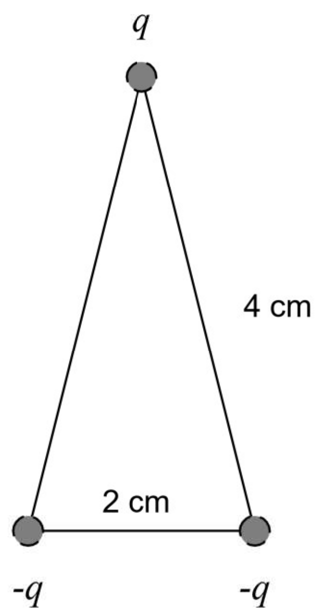


Figure 7

Calculate the electric potential at the midpoint of the base, taking q=7.00 μC.
a. -14.2 mV
b. 11.0 mV
c. 14.2 mV
d. -11.0mV

[28] An insulating rod having a linear charge density = 40.0 μC/m and linear mass density 0.100 kg/m is released from rest in a uniform electric field E=100 V/m directed perpendicular to the rod (Fig. 8).

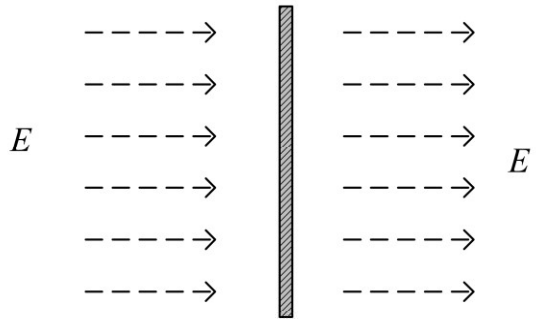


Figure 8

(a) Determine the speed of the rod after it has traveled 2.00 m. (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod?

a. (a) 0.200 m/s; (b) decreases
b. (a) 0.400 m/s; (b) the same
c. (a) 0.400 m/s; (b) decreases
d. (a) 0.200 m/s; (b) increases

---

[29] A spherical conductor has a radius of 14.0 cm and a charge of 26.0μC. Calculate the electric field and the electric potential at r = 50.0 cm from the center.

a. 9.35×10^5 N/C, 1.67 mV
b. 1.19×10^7 N/C, 0.468 mV
c. 9.35×10^5 N/C, 0.468 mV
d. 1.19×10^7 N/C, 1.67 mV

---

[30] How many electrons should be removed from an initially unchanged spherical conductor of radius 0.200 m to produce a potential of 6.50 kV at the surface?

a. 1.81×10^{11}
b. 2.38×10^{15}
c. 9.04×10^{11}
d. 1.06×10^{15}

---

[31] An ion accelerated through a potential difference of 125 V experiences an increase in kinetic energy of 9.37×10^{-17} J. Calculate the charge on the ion.

a. 1.33×10^{18} C
b. 7.50×10^{-19} C
c. 1.17×10^{-14} C
d. 1.60×10^{-19} C

---

[32] How much work is done (by a battery, generator, or some other source of electrical energy) in moving Avagadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

a. 0.482 MJ
b. 0.385 MJ
c. 1.35 MJ
d. 0.867 MJ

---

[33] At a certain distance from a point charge, the magnitude of the electric field is 600 V/m and the electric potential is -4.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

a. (a) 0.150 m; (b) 0.445 μC
b. (a) 0.150 m; (b) -1.50 μC
c. (a) 6.67 m; (b) 2.97 μC
d. (a) 6.67 m; (b) -2.97 μC

---

[34] An electron moving parallel to the x-axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point x = 2.00 cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

a. -38.9 V, the origin
b. 19.5 V, x
c. 38.9 V, x
d. -19.5 V, the origin

Solution of the multiple choice questions

Q. No.	Answer	Q. No.	Answer
1	b	18	b
2	c	19	a
3	a	20	a
4	d	21	b
5	d	22	d
6	d	23	c
7	a	24	c
8	b	25	a
9	b	26	c
10	c	27	d
11	d	28	b
12	a	29	c
13	c	30	c
14	b	31	b
15	c	32	c
16	d	33	d
17	c	34	a

Capacitors and Capacitance

المكثف الكهربي والسعة الكهربية

يعتبر هذا الفصل تطبيقاً على المفاهيم الأساسية للكهربية الساكنة، حيث سنركز على التعرف على خصائص المكثفات Capacitors وهي من الأجهزة الكهربية التي لا تخلو منها أية دائرة كهربية. ويعد المكثف بمثابة مخزن للطاقة الكهربية. والمكثف عبارة عن موصلين يفصل بينهما مادة عازلة.

6.1 Capacitor

A capacitor consists of two conductors separated by an insulator Figure 6.1. The capacitance of the capacitor depends on the geometry of the conductors and on the material separating the charged conductors, called dielectric that is an insulating material. The two conductors carry equal and opposite charge $+q$ and $-q$.

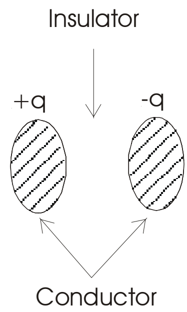


Figure 6.1

6.2 Definition of capacitance

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them as shown in Figure 6.2.

$$C = \frac{q}{V} \qquad (6.1)$$

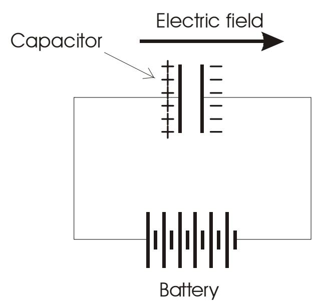


Figure 6.2

The capacitance C has a unit of C/v, which is called *farad F*

$$F = C/v$$

The farad is very big unit and hence we use submultiples of farad

$$1\mu F = 10^{-6} F$$

$$1 nF = 10^{-9} F$$

$$1 pF = 10^{-12} F$$

The capacitor in the circuit is represented by the symbol shown in Figure 6.3.

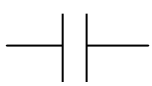


Figure 6.3

6.3 Calculation of capacitance

The most common type of capacitors are:-

- Parallel-plate capacitor
- Cylindrical capacitor
- Spherical capacitor

We are going to calculate the capacitance of parallel plate capacitor using the information we learned in the previous chapters and make use of the equation (6.1).

6.3.1 Parallel plate capacitor

Two parallel plates of equal area A are separated by distance d as shown in figure 6.4 bellow. One plate charged with $+q$, the other $-q$.

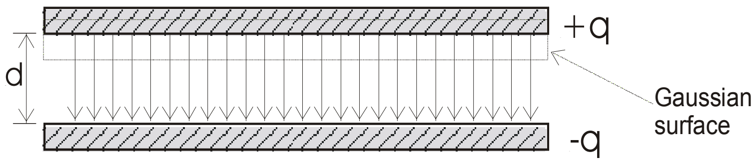


Figure 6.4

The capacitance is given by $C = \dfrac{q}{V}$

First we need to evaluate the electric field E to workout the potential V.

Using gauss law to find E, the charge per unit area on either plate is

$$\sigma = q/A. \qquad (6.2)$$

$$\therefore E = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A} \qquad (6.3), (4.9)$$

The potential difference between the plates is equal to Ed, therefore

$$V = Ed = \frac{qd}{\varepsilon_o A} \qquad (6.4)$$

The capacitance is given by

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_o A} \qquad (6.5)$$

$$\therefore C = \frac{\varepsilon_o A}{d} \qquad (6.6)$$

Notice that the capacitance of the parallel plates capacitor is depends on the geometrical dimensions of the capacitor.

The capacitance is proportional to the area of the plates and inversely proportional to distance between the plates.

المعادلة (6.6) تمكننا من حساب سعة المكثف من خلال الأبعاد الهندسية له، حيث أن سعة المكثف تتناسب طردياً مع المساحة المشتركة بين اللوحين وعكسياً مع المسافة بين اللوحين.

Example 6.1

An air-filled capacitor consists of two plates, each with an area of 7.6cm^2, separated by a distance of 1.8mm. If a 20V potential difference is applied to these plates, calculate,

(a) the electric field between the plates,
(b) the surface charge density,
(c) the capacitance, and
(d) the charge on each plate.

Solution

(a) $E = \dfrac{V}{d} = \dfrac{20}{1.8\times10^{-3}} = 1.11\times10^{4}\ V/m$

(b) $\sigma = \varepsilon_o E = (8.85\times10^{-12})(1.11\times10^{4}) = 9.83\times10^{-8}\ C/m^2$

(c) $C = \dfrac{\varepsilon_o A}{d} = \dfrac{(8.85\times10^{-12})(7.6\times10^{-4})}{1.8\times10^{-3}} = 3.74\times10^{-12}\ F$

(d) $q = CV = (3.74\times10^{-12})(20) = 7.48\times10^{-11}\ C$

6.3.2 Cylindrical capacitor

In the same way we can calculate the capacitance of cylindrical capacitor, the result is as follow

$$C = \frac{2\pi\varepsilon_o l}{\ln(b/a)} \qquad (6.7)$$

Where l is the length of the cylinder, a is the radius of the inside cylinder, and b the radius of the outer shell cylinder.

6.3.3 Spherical Capacitor

In the same way we can calculate the capacitance of spherical capacitor, the result is as follow

$$C = \frac{4\pi\varepsilon_o ab}{b-a} \qquad (6.8)$$

Where a is the radius of the inside sphere, and b is the radius of the outer shell sphere.

Example 6.2

An air-filled spherical capacitor is constructed with inner and outer shell radii of 7 and 14cm, respectively. Calculate,

(a) The capacitance of the device,

(b) What potential difference between the spheres will result in a charge of 4μC on each conductor?

Solution

(a) $C = \frac{4\pi\varepsilon_o ab}{b-a} = \frac{(4\pi \times 8.85 \times 10^{-12})(0.07)(0.14)}{(0.14-0.07)} = 1.56 \times 10^{-11} F$

(b) $V = \frac{q}{C} = \frac{4 \times 10^{-6}}{1.56 \times 10^{-11}} = 2.56 \times 10^{5} V$

6.4 Combination of capacitors

Some times the electric circuit consist of more than two capacitors, which are, connected either in parallel or in series the equivalent capacitance is evaluated as follow

6.4.1 Capacitors in parallel:

In parallel connection the capacitors are connected as shown in figure 6.5 below where the above plates are connected together with the positive terminal of the battery, and the bottom plates are connected to the negative terminal of the battery.

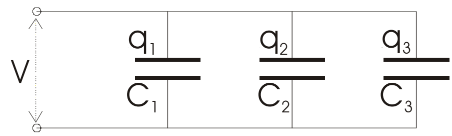


Figure 6.5

In this case the potential different across each capacitor is equal to the voltage of the battery V

$$i.e.\ V=V_1=V_2=V_3$$

The charge on each capacitor is

$$q_1 = C_1V_1; \qquad q_2 = C_2V_2; \qquad q_3 = C_3V_3$$

The total charge is

$$q = q_1 + q_2 + q_3$$
$$q = (C_1 + C_2 + C_3)V$$

$$\because C = \frac{q}{V}$$

في حالة توصيل المكثفات على التوازي يكون فرق الجهد على كل مكثف مساوياً لفرق جهد البطارية، أما الشحنة فتتوزع بنسبة سعة كل مكثف.

The Equivalent capacitance is

$$C = C_1 + C_2 + C_3 \qquad (6.9)$$

6.4.2 Capacitors in series:

In series connection the capacitors are connected as shown in figure 6.6 below where the above plates are connected together with the positive

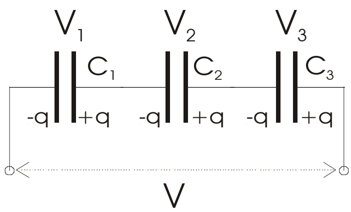


Figure 6.6

In this case the magnitude of the charge must be the same on each plate with opposite sign

$$i.e.\ q=q_1=q_2=q_3$$

The potential across each capacitor is

$$V_1 = q/C_1; \qquad V_2 = q/C_2; \qquad V_3 = q/C_3$$

The total potential V is equal the sum of the potential across each capacitor

$$V = V_1 + V_2 + V_3$$

$$V = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$C = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

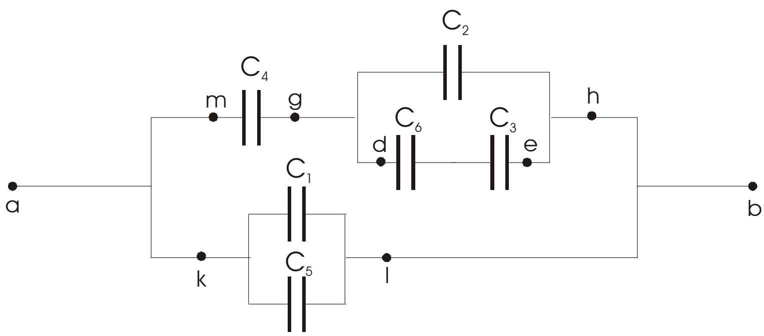
في حالة توصيل المكثفات على التوالي فإن الشحنة تتوزع على كل مكثف بشكل متساو وتساوي الشحنة الكلية. أما مجموع فروق الجهد على كل مكثف يساوي فرق جهد البطارية.

The Equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (6.10)$$

Example 6.3

Find the equivalent capacitance between points a and b for the group of capacitors shown in figure 6.7 . C_1=1μF, C_2=2μF, C_3=3μF, C_4=4μF, C_5=5μF, and C_6=6μF.



(i)

Figure 6.7

Solution

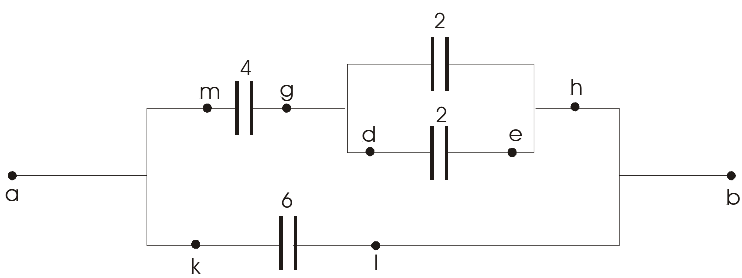
First the capacitor C_3 and C_6 are connected in series so that the equivalent capacitance C_{de} is

$$\frac{1}{C_{de}} = \frac{1}{6} + \frac{1}{3};\Rightarrow C_{de} = 2\mu F$$

Second C_1 and C_5 are connected in parallel

C_{kl}=1+5=6μF

The circuit become as shown below



(ii)

Continue with the same way to reduce the circuit for the capacitor C_2 and C_{de} to get C_{gh}=4μF

4.5 Energy stored in a charged capacitor (in electric field)

If the capacitor is connected to a power supply such as battery, charge will be transferred from the battery to the plates of the capacitor. This is a charging process of the capacitor which mean that the battery perform a work to store energy between the plates of the capacitor.

Consider uncharged capacitor is connected to a battery as shown in figure 6.8, at start the potential across the plates is zero and the charge is zero as well.

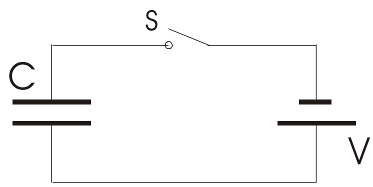


Figure 6.8

If the switch S is closed then the charging process will start and the potential across the capacitor will rise to reach the value equal the potential of the battery V in time t (called charging time).

بعد إغلاق المفتح S تستمر عملية شحن المكثف حتى يصبح فرق الجهد بين لـ وحي المكـ ف مساوياً لفرق جهد البطارية.

Suppose that at a time t a charge $q(t)$ has been transferred from the battery to capacitor. The potential difference $V(t)$ across the capacitor will be $q(t)/C$. For the battery to transferred another amount of charge dq it will perform a work dW

$$dW = Vdq = \frac{q}{C}dq \qquad (6.11)$$

The total work required to put a total charge Q on the capacitor is

$$W = \int dW = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C} \qquad (6.12)$$

Using the equation $q=CV$

$$W = U = \frac{Q^2}{2C} \qquad (6.13)$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2 \qquad (6.14)$$

The energy per unit volume u (energy density) in parallel plate capacitor is the total energy stored U divided by the volume between the plates Ad

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} \qquad (6.15)$$

For parallel plate capacitor $C = \frac{\varepsilon_o A}{d}$

$$u = \frac{\varepsilon_o}{2}\left(\frac{V}{d}\right)^2 \qquad (6.16)$$

$$u = \frac{1}{2}\varepsilon_o E^2 \qquad (6.17)$$

Therefore the electric energy density is proportional with square of the electric field.

لاحظ هنا أن الطاقة الكهربية المخزنة بين لوحي المكثف يمكن التعبير عنها باستخدام الطاقة الكلية U أو من خلال كثافة الطاقة u. الطاقة الكلية تساوي كثافة الطاقة في الحجم المحصور بين لوحي المكثف.

المعادلتان رقم (6.14)&(6.17) توضحان عنوان هذا الموضوع وهو الطاقة المخزنة في المكثف أو في المجال الكهربي.

Example 6.5

Three capacitors of 8μF, 10μF and 14μF are connected to a battery of 12V. How much energy does the battery supply if the capacitors are connected (a) in series and (b) in parallel?

Solution

(a) For series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{8} + \frac{1}{10} + \frac{1}{14}$$

This gives

$$C = 3.37\ \mu\text{F}$$

Then the energy U is

$$U = \frac{1}{2} C V^2$$

$$U = 1/2\ (3.37\times10^{-6})\ (12)^2 = 2.43\times10^{-4}\text{J}$$

(b) For parallel combination

$$C = C_1 + C_2 + C_3$$

$$C = 8+10+14=32\mu\text{F}$$

The energy U is

$$U = 1/2\ (32\times10^{-6})\ (12)^2 = 2.3\times10^{-3}\text{J}$$

Example 6.6

A capacitor C_1 is charged to a potential difference V_o. This charging battery is then removed and the capacitor is connected as shown in figure 6.9 to an uncharged capacitor C_2,

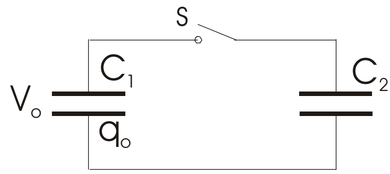


Figure 6.9

(a) What is the final potential difference V_f across the combination?

(b) What is the stored energy before and after the switch S is closed?

Solution

(a) The original charge q_o is shared between the two capacitors since they are connected in parallel. Thus

$$q_o = q_1 + q_2$$

$$q = CV$$

$$C_1 V_o = C_1 V_f + C_2 V_f$$

$$V_f = V_o \frac{C_1}{C_1 + C_2}$$

(b) The initial stored energy is U_o

$$U_o = \tfrac{1}{2} C_1 V_o^2$$

The final stored energy $U_f = U_1 + U_2$

$$U_f = \tfrac{1}{2} C_1 V_f^2 + \tfrac{1}{2} C_2 V_f^2 = \tfrac{1}{2}(C_1 + C_2)\left(\frac{V_o C_1}{C_1 + C_2}\right)^2$$

$$U_f = \left(\frac{C_1}{C_1 + C_2}\right) U_o$$

Notice that U_f is less than U_o (Explain why)

Example 6.7

Consider the circuit shown in figure 6.10 where C_1=6μF, C_2=3μF, and V=20V. C_1 is first charged by closing switch S_1. S_1 is then opened, and the charged capacitor C_1 is connected to the uncharged capacitor C_2 by closing the switch S_2. Calculate the initial charge acquired by C_1 and the final charge on each of the two capacitors.

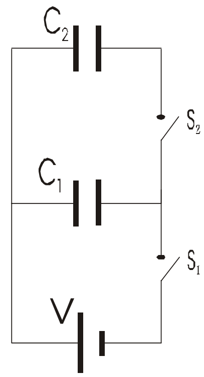


Figure 6.10

Solution

When S_1 is closed, the charge on C_1 will be

$$Q_1 = C_1 V_1 = 6\mu F \times 20V = 120\mu C$$

When S_1 is opened and S_2 is closed, the total charge will remain constant and be distributed among the two capacitors,

$$Q_1 = 120\mu C - Q_2$$

The potential across the two capacitors will be equal,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{120\mu F - Q_2}{6\mu F} = \frac{Q_2}{3\mu F}$$

Therefore,

$$Q_2 = 40\mu C$$

$$Q_1 = 120\mu C - 40\mu C = 80\mu C$$

Example 6.8

Consider the circuit shown in figure 6.11 where C_1=4μF, C_2=6μF, C_3=2μF, and V=35V. C_1 is first charged by closing switch S to point 1. S is then connected to point 2 in the circuit.

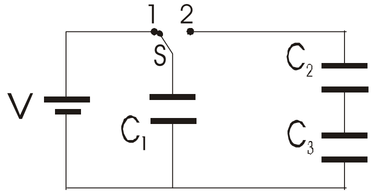


Figure 6.11

(a) Calculate the initial charge acquired by C_1,

(b) Calculate the final charge on each of the three capacitors.

(c) Calculate the potential difference across each capacitor after the switch is connected to point 2.

Solution

When switch S is connected to point 1, the potential difference on C_1 is 35V. Hence the charge Q_1 is given by

$$Q_1 = C_1 \mathrm{x} V = 4 \mathrm{x} 35 = 140 \mu C$$

When switch S is connected to point 2, the charge on C_1 will be distributed among the three capacitors. Notice that C_2 and C_3 are connected in series, therefore

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{6} + \frac{1}{2} = \frac{4}{6}$$

$$C' = 1.5 \mu F$$

We know that the charges are distributed equally on capacitor connected in series, but the charges are distributed with respect to their capacitance when they are connected in parallel. Therefore,

$$Q_1 = \frac{140}{4+1.5} \times 4 = 101.8 \mu C$$

But the charge Q' on the capacitor C' is

$$Q' = 140 - 101.8 = 38.2 \mu C$$

Since C_1 and C_2 are connected in series then

$$Q_2 = Q_3 = Q' = 38.2\mu C$$

To find the potential difference on each capacitor we use the relation $V=Q/C$

Then,

$$V_1 = 25.45V$$

$$V_2 = 6.37V$$

$$V_3 = 19.1V$$

Example 6.9

Consider the circuit shown in figure 6.12 where C_1=6μF, C_2=4μF, C_3=12μF, and V=12V.

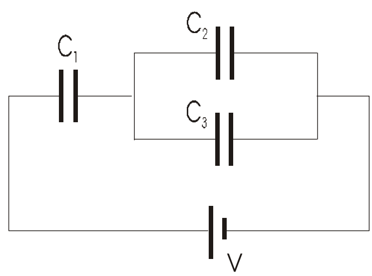


Figure 6.12

(a) Calculate the equivalent capacitance,

(b) Calculate the potential difference across each capacitor.

(c) Calculate the charge on each of the three capacitors.

Solution

C_2 and C_3 are connected in parallel, therefore

$$C' = C_2 + C_3 = 4 + 12 = 16\mu F$$

Now C' is connected in series with C_1, therefore the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{6} + \frac{1}{16} = \frac{11}{48}$$

$$C = 4.36\mu F$$

The total charge $Q = CV = 4.36x12 = 52.36\mu C$

The charge will be equally distributed on the capacitor C_1 and C'

$$Q_1 = Q' = Q = 52.36\mu C$$

But $Q' = C'V'$, therefore

$$V' = 52.36/16 = 3.27 \text{ volts}$$

The potential difference on C_1 is

$$V_1 = 12 - 3.27 = 8.73 \text{volts}$$

The potential difference on both C_2 and C_3 is equivalent to V' since they are connected in parallel.

$$V_2 = V_3 = 3.27 \text{volts}$$

$$Q_2 = C_2 V_2 = 13.08\mu C$$

$$Q_3 = C_3 V_3 = 39.24\mu C$$

Example 6.9

Four capacitors are connected as shown in Figure 6.13. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if V_{ab}=15V.

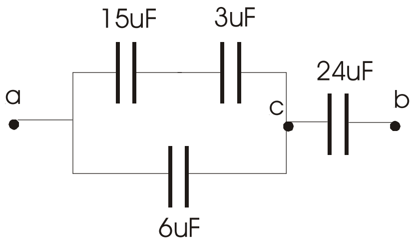
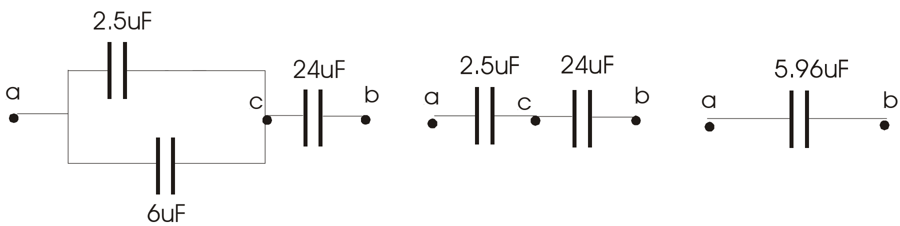


Figure 6.13

Solution

(a) We simplify the circuit as shown in the figure from (a) to (c).



(a) (b) (c)

Firs the 15μF and 3μF in series are equivalent to

$$\frac{1}{(1/15)+(1/3)} = 2.5\mu F$$

Next 2.5μF combines in parallel with 6μF, creating an equivalent capacitance of 8.5μF.

The 8.5μF and 20μF are in series, equivalent to

$$\frac{1}{(1/8.5)+(1/20)} = 5.96\mu F$$

(b) We find the charge and the voltage across each capacitor by working backwards through solution figures (c) through (a).

For the 5.96μF capacitor we have

$$Q = CV = 5.96 \times 15 = 89.5\mu C$$

In figure (b) we have, for the 8.5μF capacitor,

$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5}{8.5} = 10.5V$$

and for the 20μF in figure (b) and (a) $Q_{20} = 89.5\mu C$

$$\Delta V_{cb} = \frac{Q}{C} = \frac{89.5}{20} = 4.47V$$

Next (a) is equivalent to (b), so $\Delta V_{cb} = 4.47V$ and $\Delta V_{ac} = 10.5V$

Thus for the 2.5μF and 6μF capacitors $\Delta V = 10.5V$

$$Q_{2.5} = CV = 2.5 \times 10.5 = 26.3\mu C$$

$$Q_6 = CV = 6 \times 10.5 = 63.2\mu C$$

Therefore

$$Q_{15} = 26.3\mu C \qquad\qquad Q_3 = 26.3\mu C$$

For the potential difference across the capacitors C_{15} and C_3 are

$$\Delta V_{15} = \frac{Q}{C} = \frac{26.3}{15} = 1.75V$$

$$\Delta V_3 = \frac{Q}{C} = \frac{26.3}{3} = 8.77V$$

6.6 Capacitor with dielectric

A dielectric is a non-conducting material, such as rubber, glass or paper. Experimentally it was found that the capacitance of a capacitor increased when a dielectric material was inserted in the space between the plates. The ratio of the capacitance with the dielectric to that without it called the dielectric constant κ of the material.

$$\kappa = \frac{C}{C_o} \qquad (6.18)$$

In figure 6.14 below two similar capacitors, one of them is filled with dielectric material, and both are connected in parallel to a battery of potential V. It was found that the charge on the capacitor with dielectric is larger than the on the air filled capacitor, therefore the $C_d > C_o$, since the potential V is the same on both capacitors.

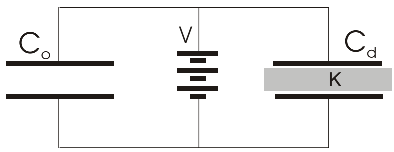


Figure 6.14

If the experiment repeated in different way by placing the same charge Q_o on both capacitors as shown in figure 6.15. Experimentally it was shown that $V_d < V_o$ by a factor of $1/\kappa$.

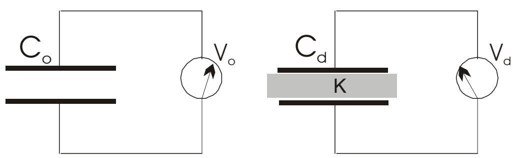


Figure 6.15

$$V_d = \frac{V_o}{\kappa} \qquad (6.19)$$

Since the charge Q_o on the capacitors does not change, then

$$C = \frac{Q_o}{V_d} = \frac{Q_o}{V_o/\kappa} = \kappa\frac{Q_o}{V_o} \qquad (6.20)$$

For a parallel plate capacitor with dielectric we can write the capacitance.

$$C = \kappa\frac{\varepsilon_o A}{d} \qquad (6.21)$$

Example 6.10

A parallel plate capacitor of area *A* and separation *d* is connected to a battery to charge the capacitor to potential difference V_o. Calculate the stored energy before and after introducing a dielectric material.

Solution

The energy stored before introducing the dielectric material,

$$U_o = \tfrac{1}{2}C_o V_o^{\,2}$$

The energy stored after introducing the dielectric material,

$$C = \kappa C_o \qquad \text{and} \qquad V_d = \frac{V_o}{\kappa}$$

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\kappa C_o\left(\frac{V_o}{\kappa}\right)^2 = \frac{U_o}{\kappa}$$

Therefore, the energy is less by a factor of 1/κ.

Example 6.11

A Parallel plate capacitor of area 0.64cm^2. When the plates are in vacuum, the capacitance of the capacitor is 4.9pF.

(a) Calculate the value of the capacitance if the space between the plates is filled with nylon (κ=3.4).

(b) What is the maximum potential difference that can be applied to the plates without causing discharge (E_{max}=14×10^{6}V/m)?

Solution

(a) $C = \kappa C_o = 3.4\times 4.9 = 16.7\text{pF}$

(b) $V_{max} = E_{max}\times d$

To evaluate d we use the equation

$$d = \frac{\varepsilon_o A}{C_o} = \frac{8.85\times 10^{-12}\times 6.4\times 10^{-5}}{4.9\times 10^{-12}} = 1.16\times 10^{-4}\,m$$

$$V_{max} = 1\times 10^{6}\times 1.16\times 10^{-4} = 1.62\times 10^{3}\ \text{V}$$

Example 6.12

A parallel-plate capacitor has a capacitance C_o in the absence of dielectric. A slab of dielectric material of dielectric constant κ and thickness *d*/3 is inserted between the plates as shown in Figure 6.16. What is the new capacitance when the dielectric is present?

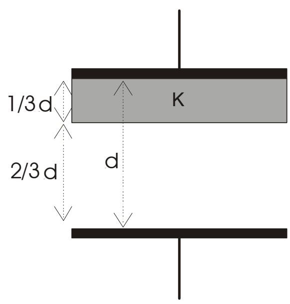


Figure 6.16

Solution

We can assume that two parallel plate capacitor are connected in series as shown in figure 6.17,

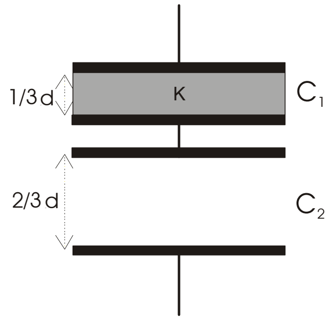


Figure 6.17

$$C_1 = \frac{\kappa\varepsilon_o A}{d/3} \qquad \text{and} \qquad C_2 = \frac{\varepsilon_o A}{2d/3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa\varepsilon_o A} + \frac{2d/3}{\varepsilon_o A}$$

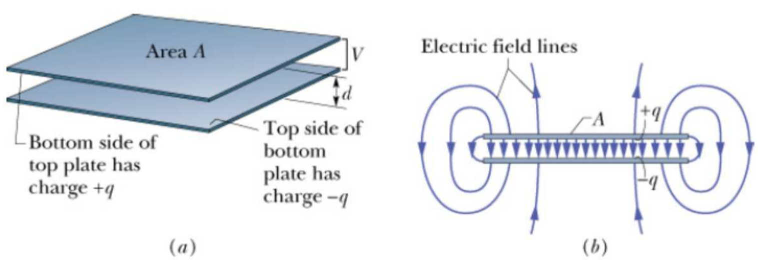
$$\frac{1}{C} = \frac{d}{3\varepsilon_o A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\varepsilon_o A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\varepsilon_o A}{d} \qquad \Rightarrow \qquad C = \left(\frac{3\kappa}{2\kappa+1}\right)C_o$$

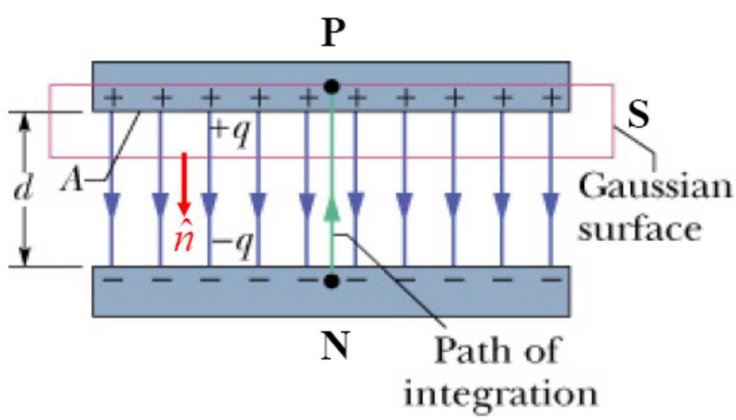
Capacitance (F)

1 F = 1 C/V
1 μF = 10^{-6} F (μ: micro)
1 nF = 10^{-9} F (n: nano)
1 pF = 10^{-12} F (p: pico)
1 fF = 10^{-15} F (f: femto)
1 aF = 10^{-18} F (a: atto)

1. Parallel plate capacitance



Parallel Plate Capacitor



$$V = Ed$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The capacitance C is defined by

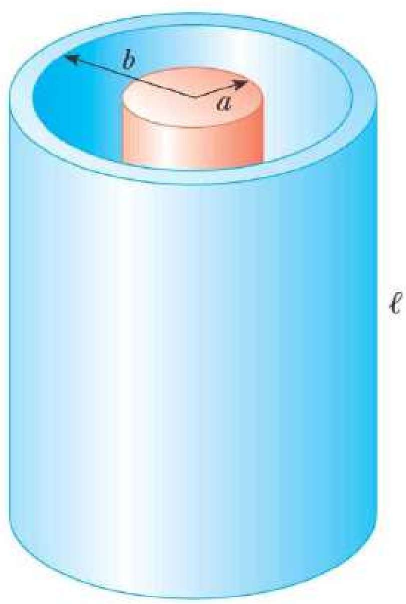
$$C = \frac{Q}{V} = \frac{\varepsilon_0 AE}{Ed} = \frac{\varepsilon_0 A}{d}$$ (parallel-plate capacitor)

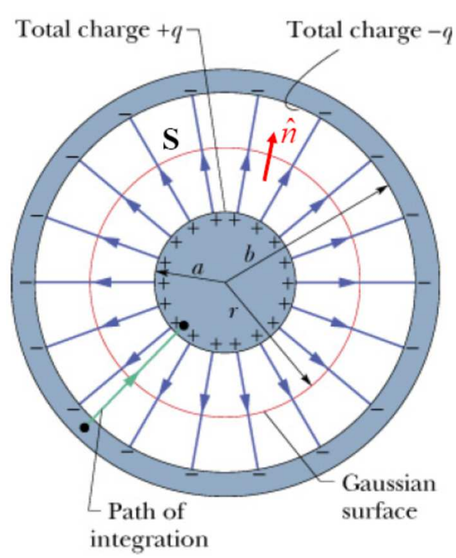
((**Note**)) **Example**

$A = 25m \times 5cm = 25 \times 0.05 m^2$, $d = 0.01mm = 10^{-5} m$

$$C = 1.11 \mu F$$

2. Cylindrical capacitor





$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

$$V_{ba} = -\int_a^b \boldsymbol{E}\cdot d\boldsymbol{r} = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\cdot\hat{r}dr$$
$$= -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}dr = -\frac{\lambda}{2\pi\varepsilon_0}\ln(\frac{b}{a})$$
$$= -\frac{1}{2\pi\varepsilon_0}\frac{q}{L}\ln(\frac{b}{a})$$

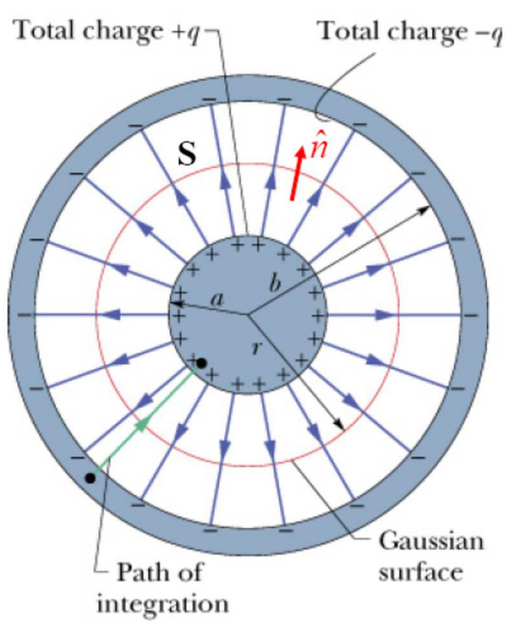
Since V_{ba}<0 (the higher potential at $r = a$ and the lower potential ar $r = b$), we put

$V_{ba} = -V$ (V>0).

The capacitance C is given by

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 V}{V}\frac{L}{\ln(\frac{b}{a})} = 2\pi\varepsilon_0\frac{L}{\ln(\frac{b}{a})},$$ (cylindrical capacitor).

3. Spherical capacitance

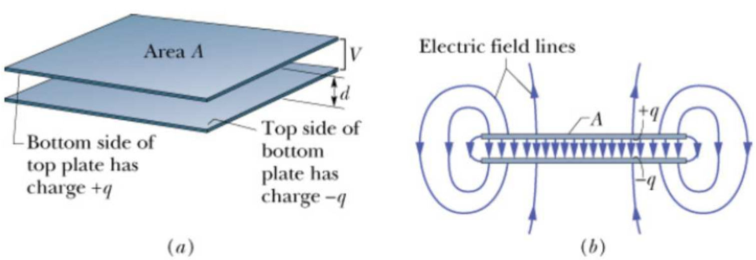


$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$$

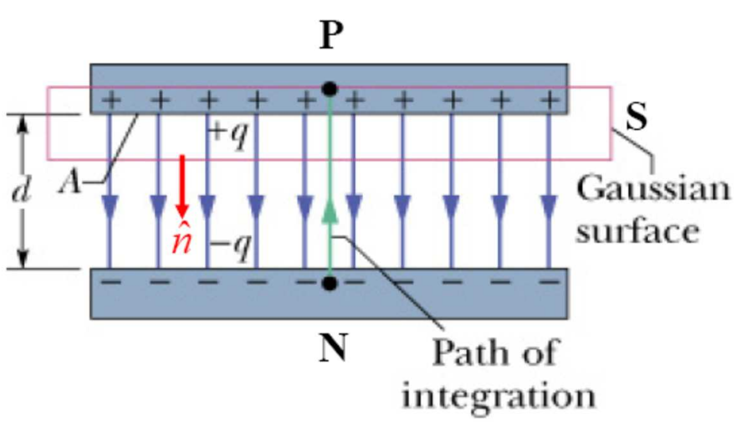
Capacitance (F)

1 F = 1 C/V
1 μF = 10^{-6} F (μ: micro)
1 nF = 10^{-9} F (n: nano)
1 pF = 10^{-12} F (p: pico)
1 fF = 10^{-15} F (f: femto)
1 aF = 10^{-18} F (a: atto)

1. Parallel plate capacitance



Parallel Plate Capacitor



$$V = Ed$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The capacitance C is defined by

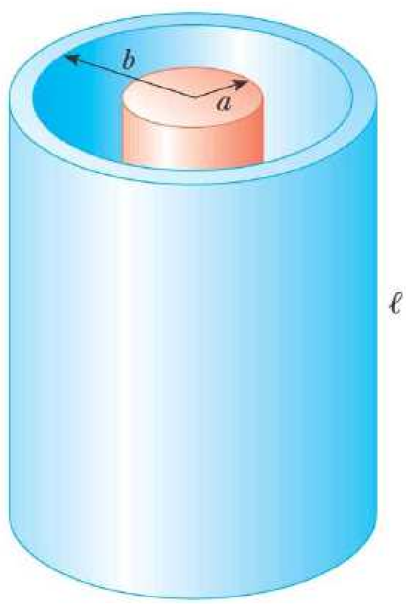
$$C = \frac{Q}{V} = \frac{\varepsilon_0 AE}{Ed} = \frac{\varepsilon_0 A}{d}$$ (parallel-plate capacitor)

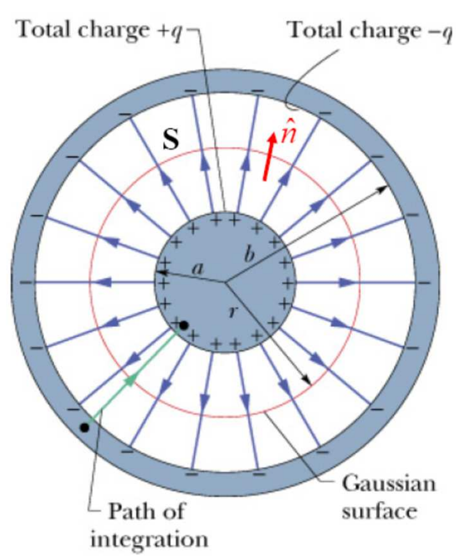
((**Note**)) **Example**

$A = 25m \times 5cm = 25 \times 0.05 m^2$, $d = 0.01mm = 10^{-5} m$

$C = 1.11 \mu F$

2. Cylindrical capacitor





$$\boldsymbol{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

$$V_{ba} = -\int_a^b \boldsymbol{E}\cdot d\boldsymbol{r} = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\cdot\hat{r}dr$$
$$= -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}dr = -\frac{\lambda}{2\pi\varepsilon_0}\ln(\frac{b}{a})$$
$$= -\frac{1}{2\pi\varepsilon_0}\frac{q}{L}\ln(\frac{b}{a})$$

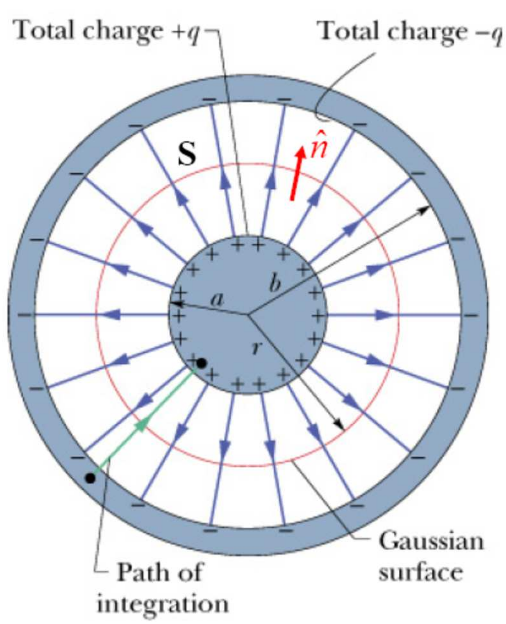
Since V_{ba}<0 (the higher potential at $r = a$ and the lower potential ar $r = b$), we put

$V_{ba} = -V$ (V>0).

The capacitance C is given by

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 V}{V}\frac{L}{\ln(\frac{b}{a})} = 2\pi\varepsilon_0\frac{L}{\ln(\frac{b}{a})},$$ (cylindrical capacitor).

3. Spherical capacitance



$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$$

$$V_{ba} = -\int_a^b \boldsymbol{E}\cdot d\boldsymbol{r} = -\int_a^b \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}\cdot\hat{r}dr$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}dr = \frac{q}{4\pi\varepsilon_0}[\frac{1}{r}]_a^b = -\frac{q}{4\pi\varepsilon_0}(\frac{1}{a}-\frac{1}{b}) = -\frac{q}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

Since $V_{ba}<0$ (the higher potential at $r = a$ and the lower potential ar $r = b$), we put

$V_{ba} = -V$ ($V>0$).

The capacitance C is given by

$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0 V}{V}\left(\frac{ab}{b-a}\right) = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$ (spherical capacitance)

4. Isolated capacitance

What is the capacitance when $a = R$ and $b \to\infty$, we have

$$C = 4\pi\varepsilon_0 R$$

where

$\varepsilon_0 = 8.854187817 \times 10^{-12}$ C^2/Nm^2

((Units))

[F] = C/V = C^2/(CV$^)$ = C^2/J = C^2/(Nm)

or

[F] = C^2/(Nm)

What is the capacitance of the Earth?

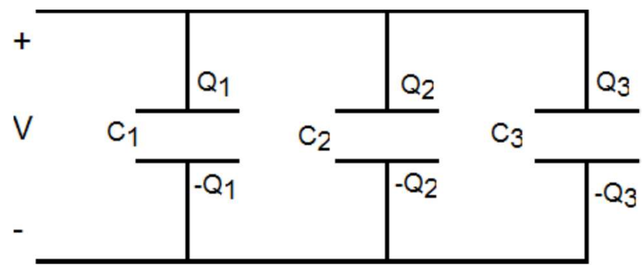
C=708.981 μF.

where the radius of the Earth (R) is

$R = 6.372 \times 10^6$ m

5. Capacitors in parallel and in series

5.1 Parallel connection



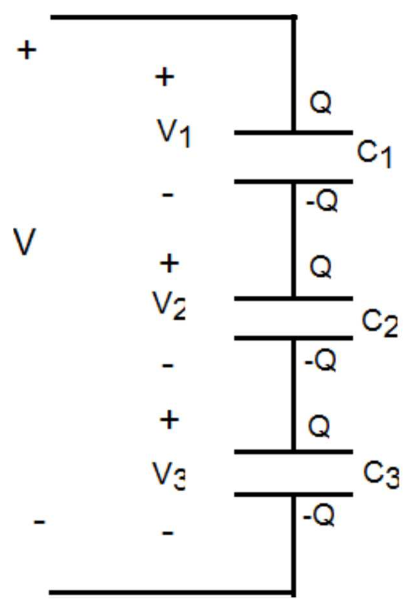
$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$
$$Q_3 = C_3 V$$

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)V$$

or

$$C = \frac{Q}{V} = C_1 + C_2 + C_3$$

5.2 Series connection



$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3$$
$$V = V_1 + V_2 + V_3$$

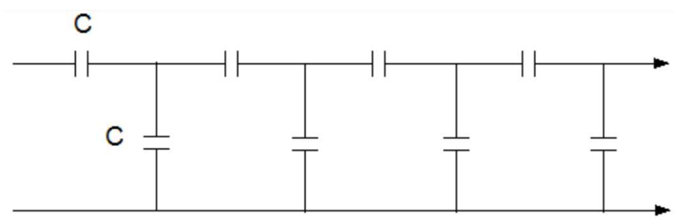
or

$$\frac{V}{Q} = \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

6. Examples

6.1 Example-1

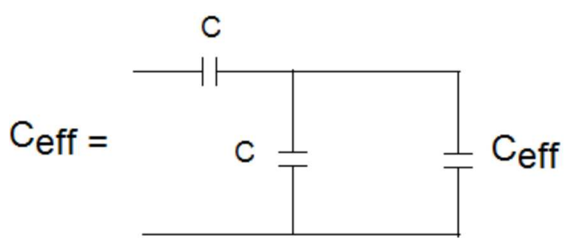
One frequency models real physical system (for example, transmission lines or nerve axons) with an infinitely repeating series of discrete circuit elements such as capacitors. Such an array is shown here. What is the capacitance between terminals X and Y for such a line, assuming it extends indefinitely? All of the capacitors are identical and have capacitance C.



(Schaum's outlines Physics for Engineering and Science, by M.E. Browne) p.281.

((**Solution**))

We assume that the effective capacitance C_{eff} is defined by the equivalent circuit given by



From this equivalent circuit, we have the following relation

$$C_{eff} = \frac{C(C + C_{eff})}{2C + C_{eff}}$$

or

$$C_{eff}^{\ 2} + CC_{eff} - C^2 = 0$$

or

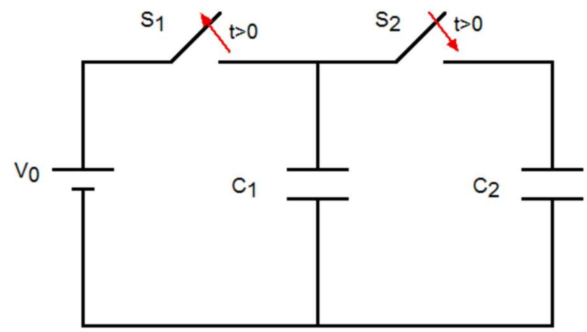
$$C_{eff} = \frac{\sqrt{5} - 1}{2} C = 0.62C$$

6.2 Example-2

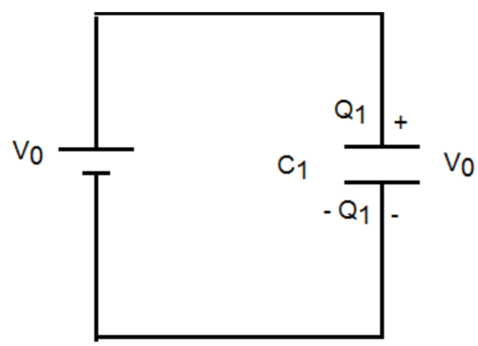
A 6 μF capacitor is charged by a 12 V battery and then disconnected. It is then connected to an uncharged 3 μF capacitor. What is the final potential difference across each capacitor?

((**Solution**))

$V_0 = 12$ V
$C_1 = 6$ μF
$C_2 = 3$ μF

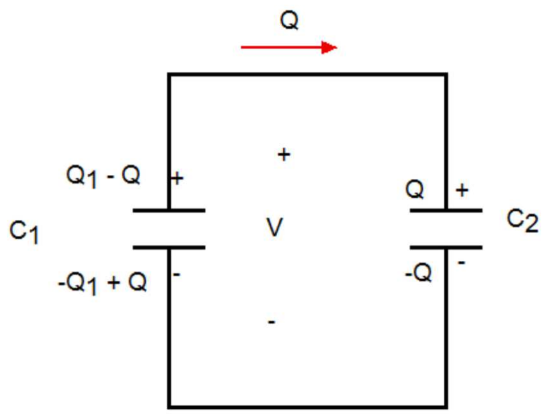


(a) t<0



$$Q_1 = C_1 V_0 == 72 \text{ μC}.$$

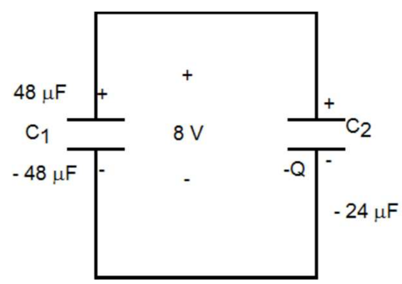
(b) t>0



$$Q_1 - Q = C_1 V$$
$$Q = C_2 V$$

or

$$Q_1 = (C_1 + C_2)V$$
$$V = \frac{Q_1}{C_1 + C_2} = \frac{C_1 V_0}{C_1 + C_2} = 8V$$

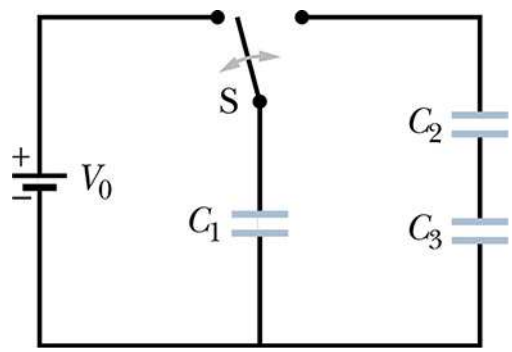


Then we have

$$V = 8V$$
$$Q = C_2 V = 24\mu C$$

7. Typical examples ((25-26))

Figure displays a 12.0 V battery and three uncharged capacitors of capacitances C_1 = 4.00 μF, C_2 = 6.00 μF, and C_3 = 3.00 μF. The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?



C_1 = 4.00 μF, C_2 = 6.00 μF, C_3 = 3.00 μF. V_0 = 12.0 V

((**Solution**))

$$Q_1 = C_1 V_0 = 4\mu F \times 12V = 48\mu C$$

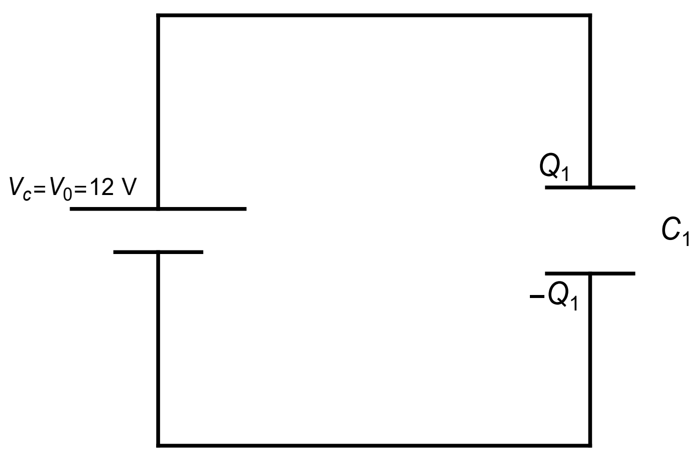
$$Q_1 - Q = C_1 V_1, \qquad Q = C_2 V_2 = C_3 V_3, \qquad V_1 = V_2 + V_3$$

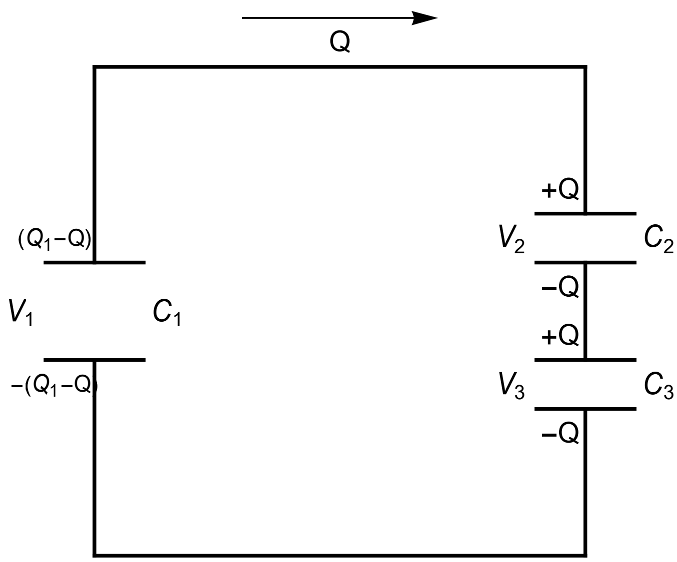
$$V_3 = \frac{C_2}{C_3} V_2 = \frac{6\mu F}{3\mu F} V_2 = 2V_2$$

From these relations we get

$$V_1 = 8\text{V}, \qquad V_2 = \frac{8}{3}\text{ V}, \qquad \text{and} \qquad V_3 = \frac{16}{3}\text{ V}.$$

$$Q_2 = Q_3 = 16\,\mu\text{C}.$$





8. The Energy of capacitance

To "charge up" a capacitor, we have to remove electrons from the positive plate and carry them to the negative plate. In doing so, one fight against the electric field, which is pulling them back toward the positive conductor and pushing them away from the negative one. How much work does it take, then, to charge the capacitor up to a final amount Q? Suppose that at some intermediate stage in the process the charge on the positive plate is q, so that the potential difference is q/C. the work you must do to transport the next piece of charge, dq, is

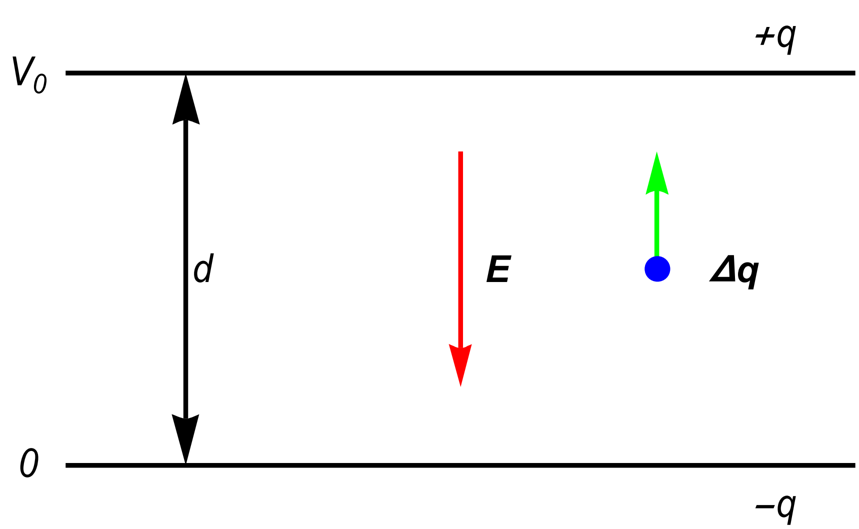


Fig. $V_0 = Ed$. $q = CV_0$

$$\Delta W = -\Delta q(Ed)$$
$$= -V_0 \Delta q$$
$$= -\frac{q}{C}\Delta q$$

where $\boldsymbol{F}$ is the force and $\boldsymbol{F} = \Delta q \boldsymbol{E}$ and $V_0 = Ed = \frac{q}{C}$ The total work necessary, then, to go from $q = 0$ to $q = Q$, is

$$W = -\int_0^Q \frac{q}{C} dq = -\frac{Q^2}{2C} = -\frac{1}{2} CV^2$$

where Q is the total charge, $Q = CV$, V is the final electric potential of the capacitor. Using the work-energy theorem, we have the potential energy U as

$$U = -W = \frac{Q^2}{2C} = \frac{CV^2}{2}$$

((Note-1)) Feynman

Recalling that the capacity of a conducting sphere (relative to infinity) is

$$C = 4\pi\varepsilon_0 R$$

where R is the radius of sphere. Thus the energy of a charged sphere is

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

((Note-2))

The energy density u is defined by

$$u = \frac{U}{Ad} = \frac{1}{Ad}\frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2$$

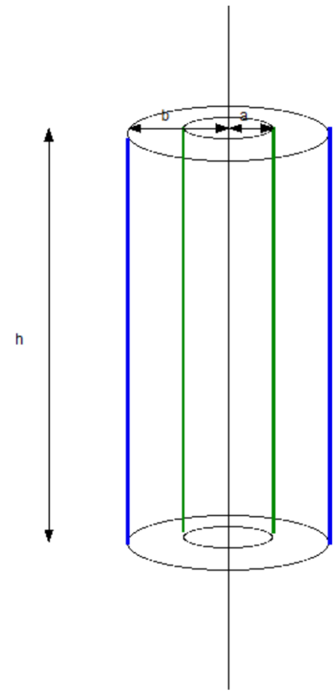
where Ad is the volume, $C = \frac{\varepsilon_0 A}{d}$, and $V = Ed$. The total energy of the capacitance can be rewritten as

$$U = \int \frac{1}{2}\varepsilon_0 \boldsymbol{E}^2 d^3\boldsymbol{r}$$

9. Example Problem ((25-68))

A cylindrical capacitor has radii a and b in Fig. Show that half the stored electric potential energy lies within a cylinder whose radius is $r = \sqrt{ab}$.

((**Solution**))



((**Solution**))

From the Gauss' theorem, we have

$$E = \frac{1}{2\pi rh}\frac{1}{\varepsilon_0}\lambda h = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The energy density is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)^2 = \frac{\lambda^2}{8\pi^2\varepsilon_0 r^2}$$

The total energy U is

$$U = \int_a^b u(2\pi r)hdr = \int_a^b \frac{1}{2}\varepsilon_0\left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)^2 (2\pi r)hdr$$
$$= \frac{\lambda^2 h}{4\pi\varepsilon_0}\int_a^b \frac{1}{r}dr = \frac{\lambda^2 h}{4\pi\varepsilon_0}\ln(\frac{b}{a})$$

U_{half} is defined as

$$U_{half} = \frac{\lambda^2 h}{4\pi\varepsilon_0} \ln(\frac{r}{a})$$

$$\frac{U_{half}}{U} = \frac{1}{2} = \frac{\ln(\frac{r}{a})}{\ln(\frac{b}{a})}$$

or

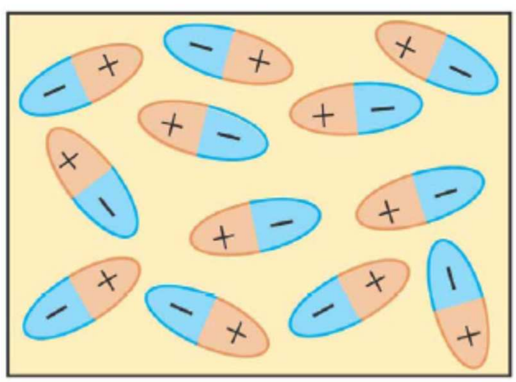
$$\ln(\frac{r^2}{a^2}) = \ln(\frac{b}{a}),$$

or

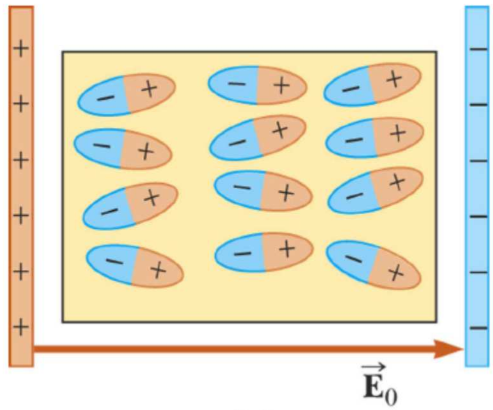
$$r = \sqrt{ab}\,.$$

10. Dielectrics in the presence of electric field: atomic view

The molecules that make up the dielectric are modeled as dipoles. The molecules are randomly oriented in the absence of an electric field.

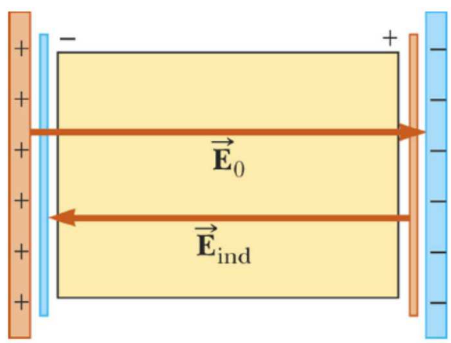


Suppose that an external electric field is applied. This produces a torque on the molecules. The molecules partially align with the electric field.



(b)

An external field can polarize the dielectric whether the molecules are polar or nonpolar. The charged edges of the dielectric act as a second pair of plates producing an induced electric field in the direction opposite the original electric field



(c)

11. Experiment (I) Charge remained constant

Walter Lewin" 8.02X MIT Physics, Electricity and Magnetism
Lecture 8

The capacitance of a set of charged parallel plates is increased by the insertion of a dielectric material.

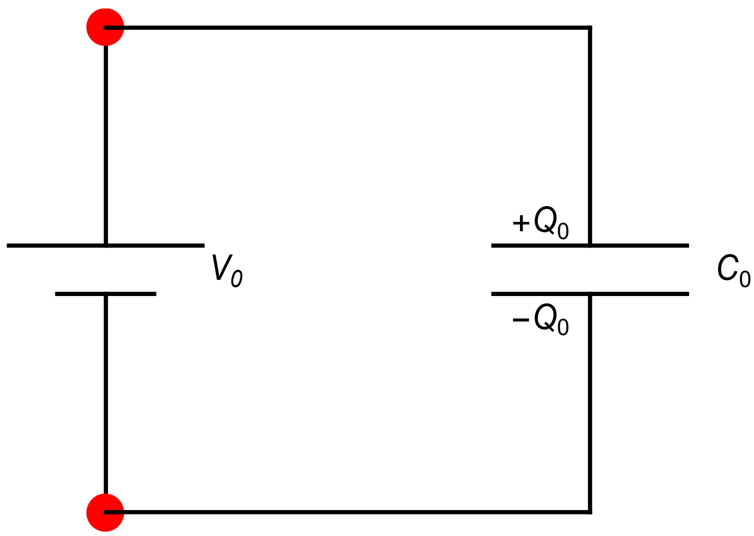
$$C_0 = \frac{\varepsilon_0 A}{d}, \qquad C = \frac{\varepsilon_0 \kappa A}{d}, \qquad \frac{C}{C_0} = \kappa \qquad \text{(dielectric constant)}$$

We discuss the physical meaning of κ using the following experiments.

(a) Step-1 (closed circuit)

The capacitance (C_0) is charged to the charge Q_0 by connecting a voltage source V_0.

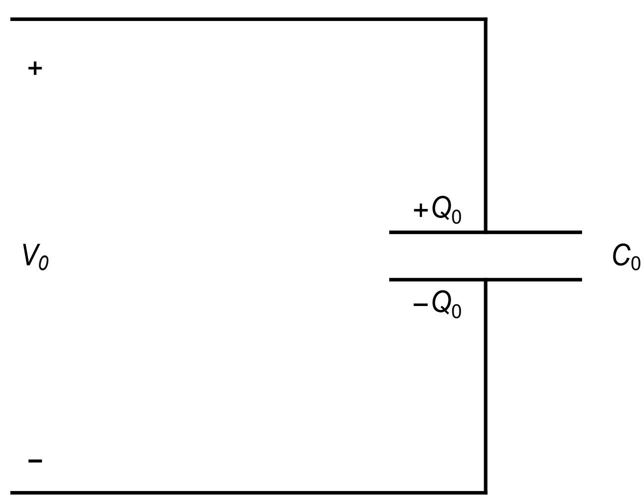
$Q_0 = C_0V_0$



(b) Step-2 (open circuit)

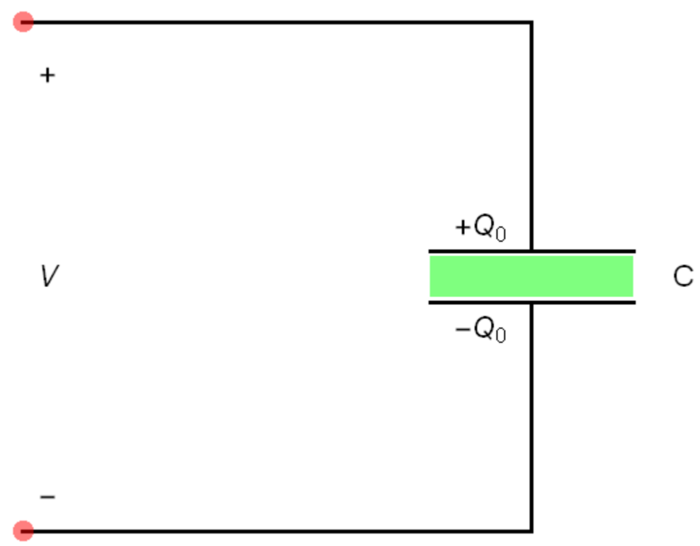
The voltage source is disconnected from the circuit. The charge remains unchanged during this process.

$Q_0 = C_0V_0$



(c) Step-3 (open circuit)

A dielectric material is inserted into the space between two electrodes of the capacitance. The capacitance changes from $C_0 \to C$. The voltage across the capacitance changes.



The free charge Q_0 remains unchanged, while the voltage across the capacitance changes from V_0 to V,

$$Q_0 = CV$$

or

$$Q_0 = C_0 V_0 = CV$$

Suppose that

$$\frac{C}{C_0} = \kappa$$

(The dielectric medium is inserted into the interlamellar space of the capacitance)

Then we have

$$\frac{V}{V_0} = \frac{C_0}{C} = \frac{1}{\kappa} \qquad V = \frac{1}{\kappa} V_0$$

Since $V_0 = E_0 d$ and $V = Ed$

$$E = \frac{1}{\kappa} E_0$$

where d is the separation distance between two electrodes of the capacitance. We note that

$$C = \frac{C_0 V_0}{V} = \frac{Q_0}{V}$$

((**Note**))

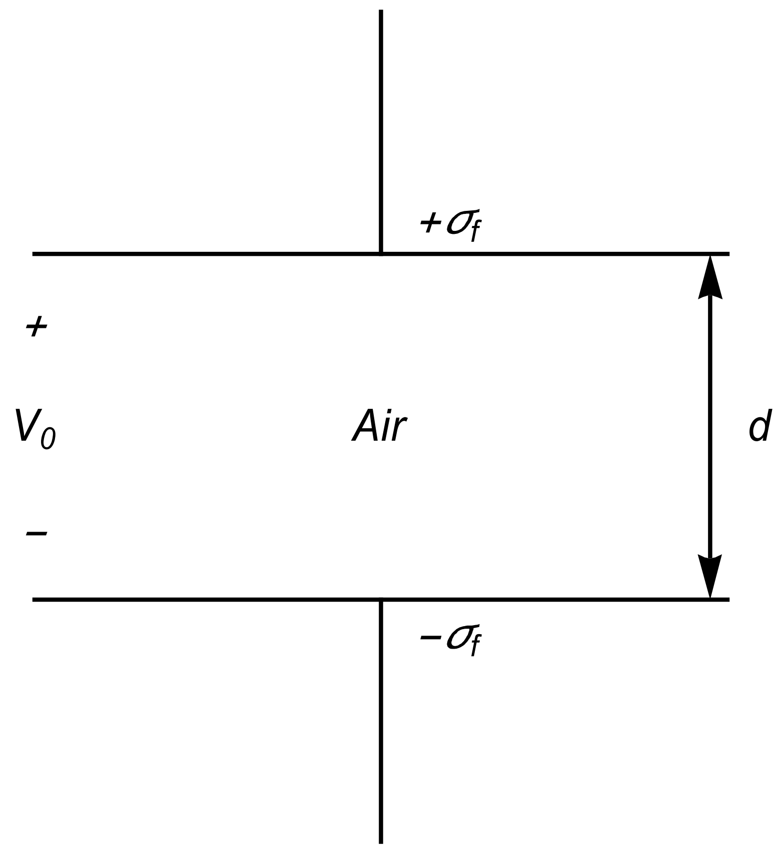
$$E = \frac{1}{\kappa} E_0 \qquad\qquad V = \frac{1}{\kappa} V_0$$

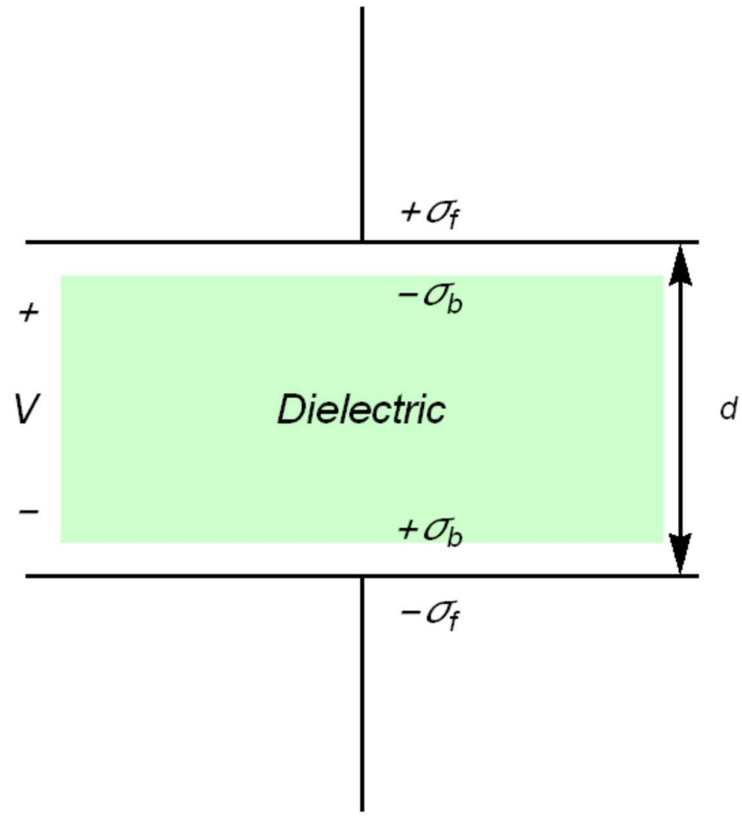
Since

$$E = \frac{\sigma_f - \sigma_b}{\varepsilon_0} = \frac{V}{d}, \qquad\qquad E_0 = \frac{\sigma_f}{\varepsilon_0} = \frac{V_0}{d}$$

we get

$$\frac{\sigma_f - \sigma_b}{\varepsilon_0} = \frac{1}{\kappa}\frac{\sigma_f}{\varepsilon_0}, \qquad \text{or} \qquad \sigma_b = (1 - \frac{1}{\kappa})\sigma_f$$



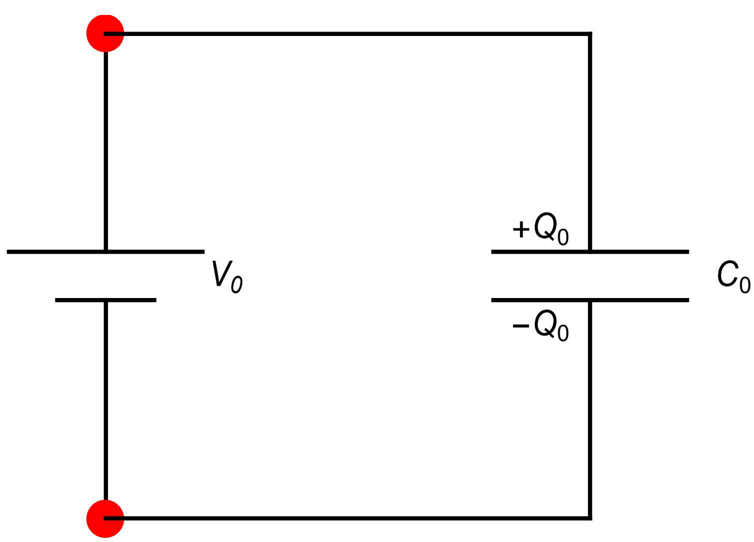


12. Experiment II Constant voltage source

(a) Step-I

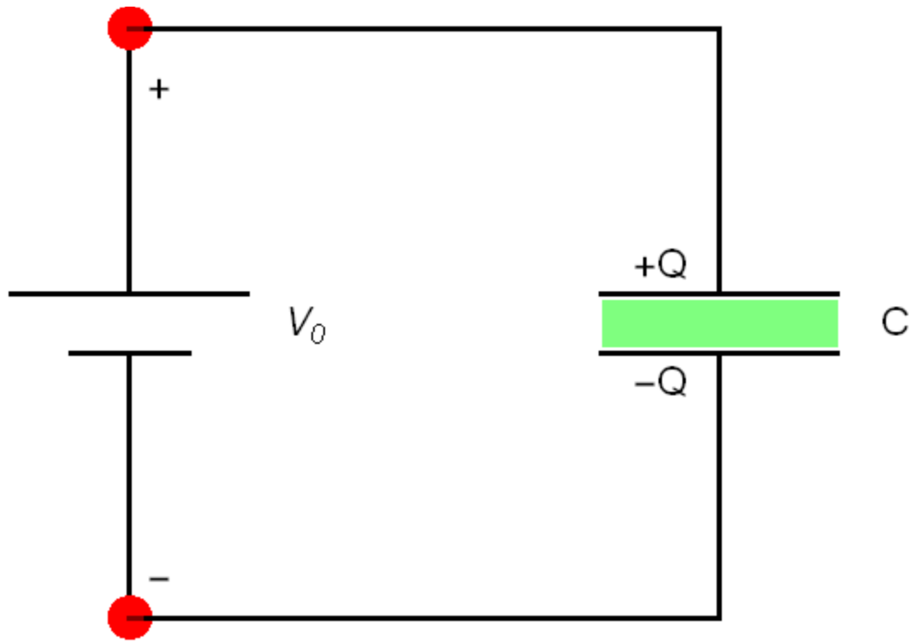
The capacitance (C_0) is charged to the charge Q_0 by connecting a voltage source V_0.

$$Q_0 = C_0 V_0$$



(b) Step-II

While the battery continues to be connected, the dielectric is inserted into a gap between two electrodes of the capacitance. While the voltage remains unchanged as V_0, the charge changes from Q_0 to Q.



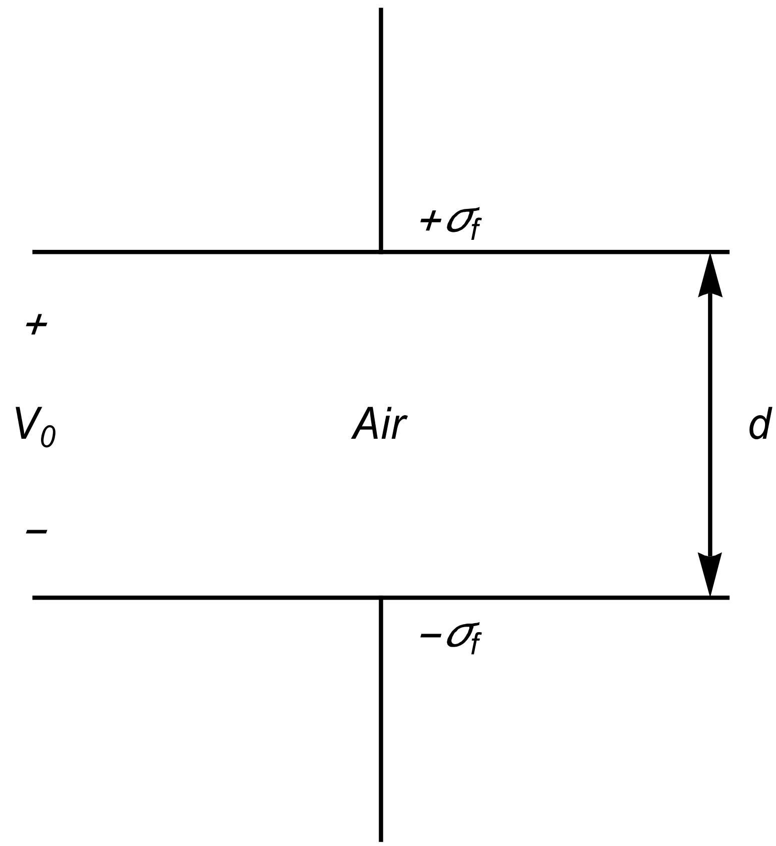
$$Q = CV_0$$

Since $C = \kappa C_0$, we have

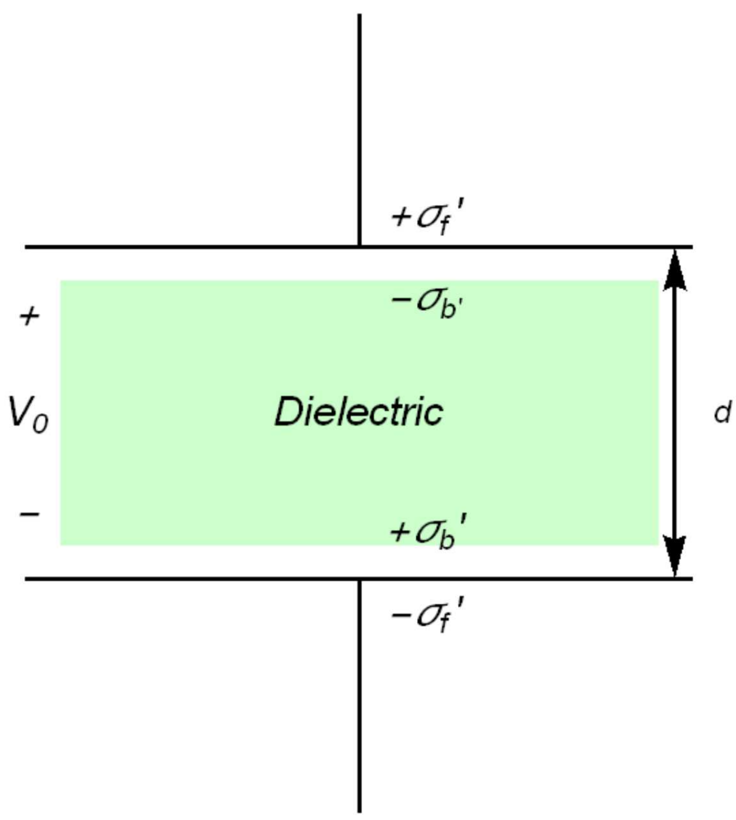
$$\frac{Q}{Q_0} = \frac{CV_0}{C_0 V_0} = \frac{C}{C_0} = \kappa, \qquad Q = \kappa Q_0$$

The electric field E remains unchanged during this process, since the applied voltage is kept constant.

((**Note**))



$C_0 V_0 = A\sigma_f$



$$CV_0 = \kappa C_0 V_0 = \kappa A \sigma_f = A(\sigma_f{}' - \sigma_P{}'),$$

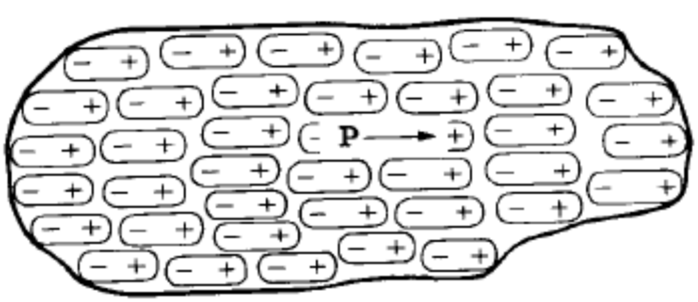
or

$$\sigma_f{}' - \sigma_P{}' = \kappa\sigma_f$$

or

$$\sigma_f{}' = \kappa\sigma_f + \sigma_P{}'$$

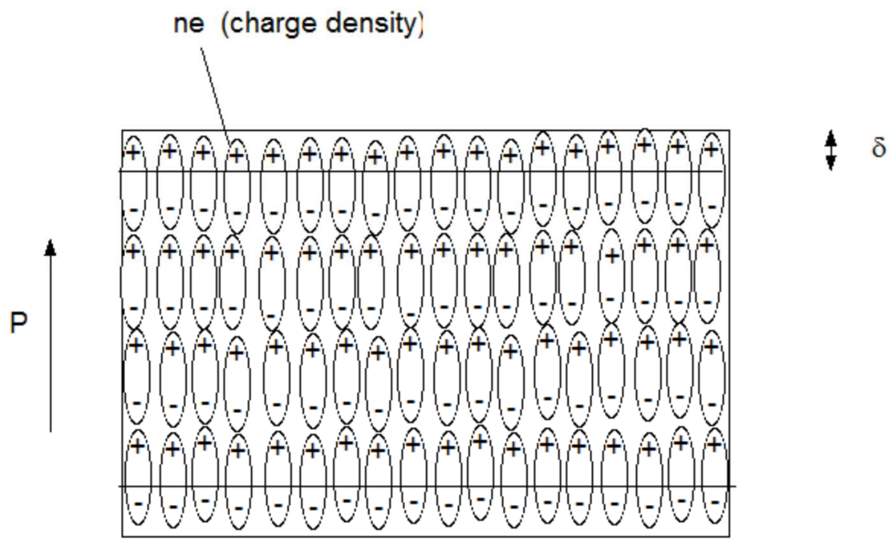
13. Polarization vector *P*



Suppose that the molecules with permanent electric dipole moments are lined neatly, all pointing the same way, and frozen in position. There are N dipoles (with electric dipole moment $\boldsymbol{p}$) per cubic meters. We shall assume that N is so large that any macroscopically small volume $d\tau$ contains quite a large number of dipoles. The total dipole strength in such a volume is $\boldsymbol{p}N d\tau$. At any point far away from this volume element compared with its size, the electric field from these particular dipoles would be practically the same if they were replaced by a single dipole moment of strength $\boldsymbol{p}N d\tau$. We shall call $\boldsymbol{p}N$ the density of polarization, and denoted it by $\boldsymbol{P}$. Then $\boldsymbol{P}d\tau$ is the dipole moment to be associated with any small volume element $d\tau$.

14. Feynman's comment on the expression of ρ_b and σ_b

Feynman's lecture on physics



We consider the above situation, where P is uniform in the above figure. We have a positive charge at the one side

$$\Delta Q = enA\delta$$

and a negative charge

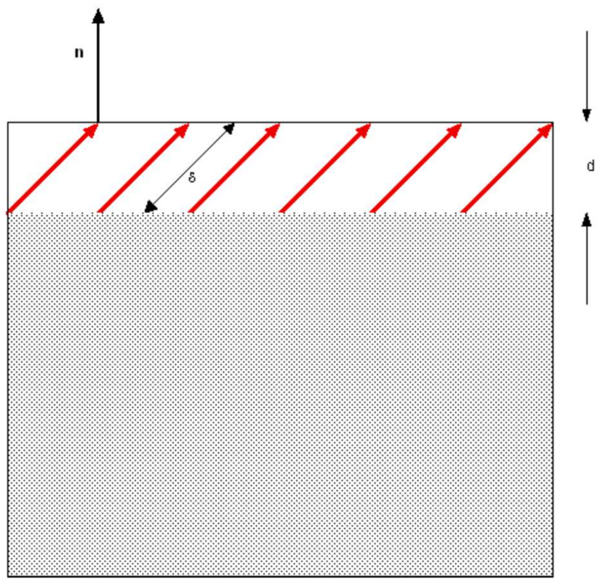
$$-\Delta Q = -enA\delta,$$

where A is the surface area, δ is the displacement, $-e$ is the electron charge, and n is the number of electrons per unit volume. From the definition, the surface charge density is given by

$$\sigma_b = \frac{\Delta Q}{A} = (e\delta)n = pn = P$$

where p (= $e\delta$) is the electric dipole moment. The vector P is the polarization vector. The magnitude P is the electric dipole moment per unit volume.

What happens to σ_b when $\boldsymbol{P}$ does not point to the direction perpendicular to the surface?

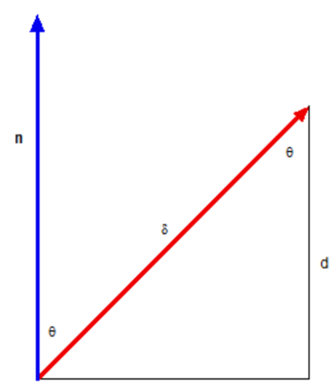


The total charge in the surface region (d) is equal to

$$\Delta Q' = enAd$$

When the angle between $\boldsymbol{P}$ and the normal unit vector $\boldsymbol{n}$ (perpendicular to the surface) is θ, the relation between d and δ is given by

$$d = \delta \cos\theta$$



Then the surface charge density is

$$\sigma_b = \frac{\Delta Q'}{A} = (ed)n = (e\delta)n\cos\theta = pn\cos\theta = P\cos\theta$$

or

$\sigma_b = \boldsymbol{P} \cdot \boldsymbol{n}$

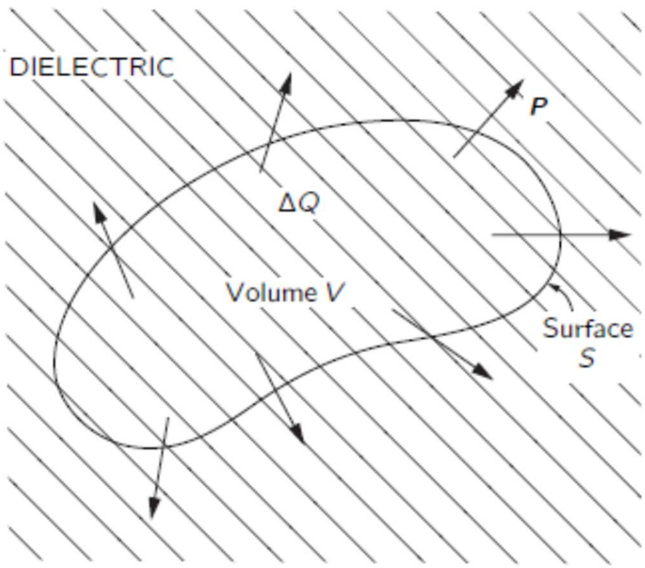
From the Gauss' law,

$$\int \nabla \cdot \boldsymbol{P} d\tau = \int \boldsymbol{P} \cdot \boldsymbol{n} da = \int \sigma_b da \,. \tag{1}$$

Since the total charge is equal to zero, we have

$$\int \rho_b d\tau + \int \sigma_b da = 0 \,, \tag{2}$$

where ρ_b is the volume charge density.



From Eqs.(1) and (2), we get

$$\int \nabla \cdot \boldsymbol{P} d\tau = -\int \rho_b d\tau$$

or

$\rho_b = -\nabla \cdot \boldsymbol{P}$

((**Note**)) We define the current density due to the polarization vector ***P*** as

$$\boldsymbol{J}_b = \frac{\partial \boldsymbol{P}}{\partial t}$$

$$\nabla \cdot \boldsymbol{J}_b = \frac{\partial}{\partial t}(\nabla \cdot \boldsymbol{P}) = -\frac{\partial}{\partial t}\rho_b, \quad \text{or} \qquad \nabla \cdot \mathbf{J}_b + \frac{\partial}{\partial t}\rho_b = 0$$

which corresponds to the continuity of the polarization current.

15. Displacement vector: Derivation of the σ_b and ρ_b from the electric potential

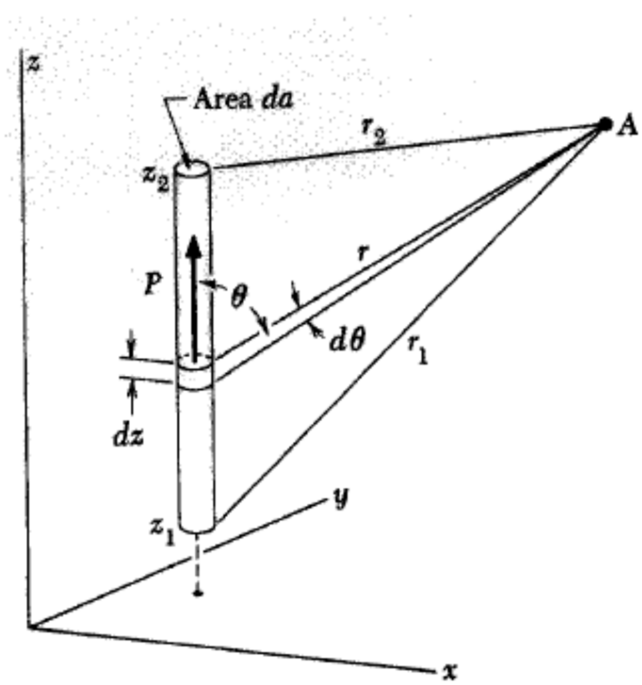
(a) 1D case

Here we also assume that there is no net charge in the system. So we have only the dipole moments to consider as sources of a distant field. The figure shows a slender column, or cylinder, of this polarized material. Its cross section is da, and it extends vertically from z_1 to z_2. The polarization density $\boldsymbol{P}$ within the column is uniform over the length and points in the positive z direction. Now we calculate the electrical potential, at some external point, of this column polarization. An element of the cylinder, of height dz, has a dipole moment $\boldsymbol{P}$dadz. It contribution to the potential at the point A can be described by

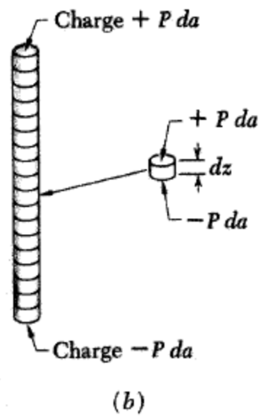
$$dV_A = \frac{1}{4\pi\varepsilon_0}\frac{(\boldsymbol{P}dadz)\cdot \boldsymbol{r}}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{Pdadz\cos\theta}{r^2}$$

The potential due to the entire column is

$$V_A = \frac{1}{4\pi\varepsilon_0}\int_{z_1}^{z_2}\frac{Pdadz\cos\theta}{r^2}$$



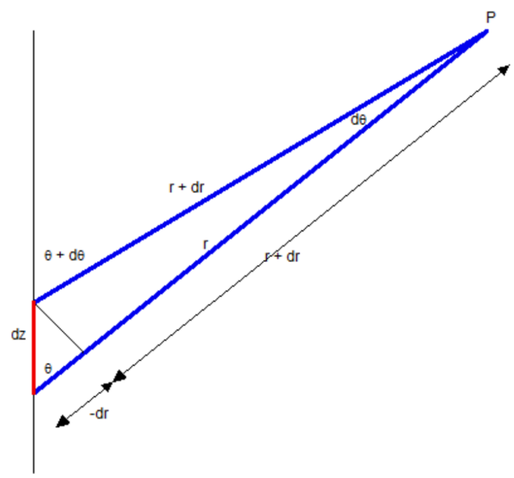
(a)



(b)

Since $dz\cos\theta = -dr$,

$$V_A = \frac{1}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{Pda(-dr)}{r^2} = \frac{Pda}{4\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$



This is precisely the same as the expression for the potential at A that would be produced by two point charges, a positive charge of magnitude Pda sitting on the top of the column at a distant r_2 from A, and a negative charge of the same magnitude at the bottom of the column. *The source consisting of a column of uniformly polarized matter is equivalent to two concentrated charges.*

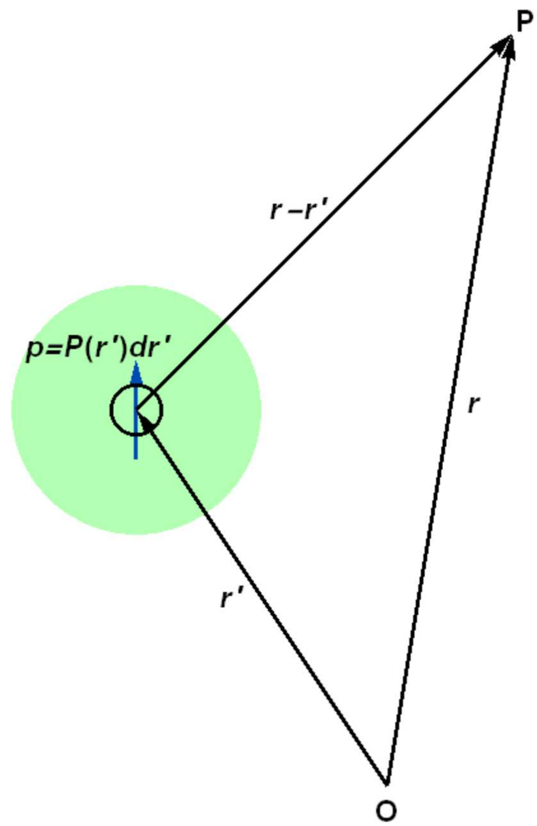
(b) General case

We consider a finite piece of dielectric material which is polarized. We define a polarization $\boldsymbol{P}(\boldsymbol{r}')$ at each point $\boldsymbol{r}'$ in the system. Each volume dτ' is characterized by an electric dipole moment $\boldsymbol{P}(\boldsymbol{r}')d\tau'$. The contribution of the electric potential at the point $\boldsymbol{r}$ from the moment $\boldsymbol{P}(\boldsymbol{r}')d\tau'$ is given by

$$dV(\boldsymbol{r}) = \frac{\boldsymbol{P}(\boldsymbol{r}')d\tau'\cdot(\boldsymbol{r}-\boldsymbol{r}')}{4\pi\varepsilon_0\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3}$$

Then the entire potential at point *r* is obtained as

$$V(\boldsymbol{r}) = \int \frac{\boldsymbol{P}(\boldsymbol{r}')\cdot(\boldsymbol{r}-\boldsymbol{r}')}{4\pi\varepsilon_0\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3}d\tau'$$



We use the formula of the vector analysis,

$$\nabla'\frac{1}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|} = \frac{\boldsymbol{r}-\boldsymbol{r}'}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3},$$

and

$$\nabla'\cdot(f\boldsymbol{A}) = f\nabla'\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\nabla' f,$$

where *f* is any scalar point function and ***A*** is an arbitrary vector point function. The prime indicates differentiation with respect to the prime coordinates. Letting ***A*** = ***P*** and

$$f = \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|}.$$

Using the relation

$$\frac{\boldsymbol{P}(\boldsymbol{r}')\cdot(\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} = \boldsymbol{P}(\boldsymbol{r}')\cdot\nabla'\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} = \nabla'\cdot(\frac{\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}) - \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|}\nabla'\cdot\boldsymbol{P}(\boldsymbol{r}'),$$

we have

$$\begin{aligned} V(\boldsymbol{r}) &= \frac{1}{4\pi\varepsilon_0}\int[\nabla'\cdot(\frac{\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}) - \frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|}\nabla'\cdot\boldsymbol{P}(\boldsymbol{r}')]d\tau' \\ &= \frac{1}{4\pi\varepsilon_0}[\oint\frac{\boldsymbol{P}(\boldsymbol{r}')\cdot\boldsymbol{n}'}{|\boldsymbol{r} - \boldsymbol{r}'|}da' + \int\frac{-\nabla'\cdot\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}d\tau'] \end{aligned},$$

where the volume integral of $\nabla'\cdot(\frac{\boldsymbol{P}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|})$ is replaced by a surface integral through the application of the Gauss' theorem and ***n'*** is the outward normal to the surface element d*a'*.

Here we define

$$\sigma_b = \boldsymbol{P}\cdot\boldsymbol{n} = P_n,$$

and

$$\rho_b = -\nabla\cdot\boldsymbol{P}.$$

The surface charge density σ_b is given by the component of the polarization $\boldsymbol{P}$ normal to the surface and the volume charge density ρ_b is a measure of the nonuniformity of the polarization $\boldsymbol{P}$ in side the system. So we have the final form of $V(\boldsymbol{r})$ as

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}[\oint\frac{\sigma_P}{|\boldsymbol{r} - \boldsymbol{r}'|}da' + \int\frac{\rho_P}{|\boldsymbol{r} - \boldsymbol{r}'|}d\tau']$$

(c) Electric displacement D

$$\varepsilon_0\nabla\cdot\boldsymbol{E} = \rho_f + \rho_P$$

with

$$\rho_P = -\nabla\cdot\boldsymbol{P}$$

where P is the polarization vector (charge per unit area). Thus we have

$$\nabla\cdot(\varepsilon_0\boldsymbol{E}+\boldsymbol{P})=\rho_f$$

or

$$\nabla\cdot\boldsymbol{D}=\rho_f$$

It is customary to give the combination $\varepsilon_0\boldsymbol{E}+\boldsymbol{P}$ a special name, the electric displacement vector and its own symbol ***D***,

$$\boldsymbol{D}=\varepsilon_0\boldsymbol{E}+\boldsymbol{P}$$

Using the Gauss's law, we have

$$\int\nabla\cdot\boldsymbol{D}\,d^3\boldsymbol{r}=\int\rho_f\,d^3\boldsymbol{r}=Q_f$$

or

$$\int\boldsymbol{D}\cdot d\boldsymbol{a}=\int\rho_f\,d^3\boldsymbol{r}=Q_f$$

We have

$$\boldsymbol{D}=\varepsilon\boldsymbol{E}$$

$$\boldsymbol{P}=\varepsilon_0\chi_e\boldsymbol{E}$$

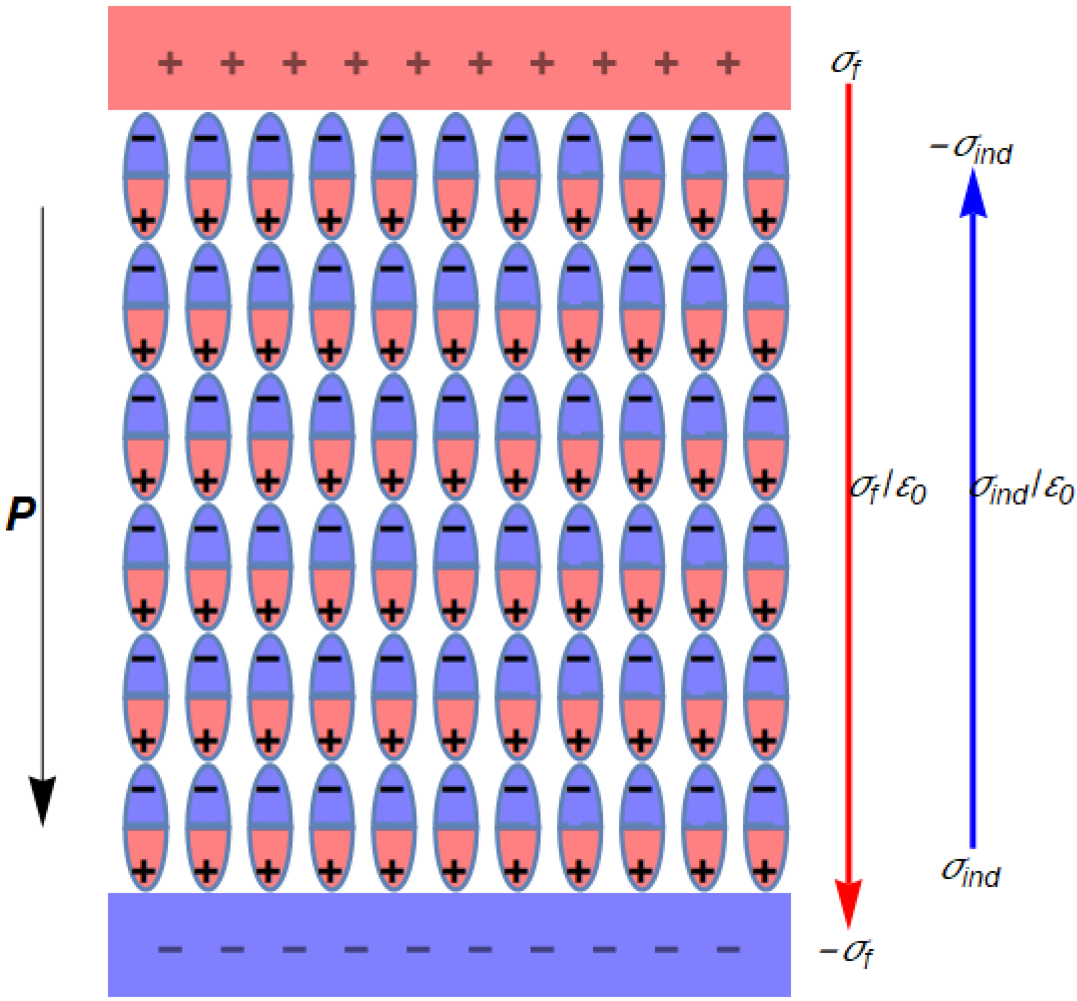
$$\varepsilon\boldsymbol{E}=\varepsilon_0\boldsymbol{E}+\varepsilon_0\chi_e\boldsymbol{E}$$

where

$$\frac{\varepsilon}{\varepsilon_0}=\varepsilon_r=1+\chi_e=\kappa$$

16 Capacitance with dielectric (I)

Here we discuss the capacitance of the dielectric.



In this figure, σ_f is the free charge. $\boldsymbol{P}$ is the polarization vector. The inductive charge $\sigma_{ind} = \sigma_b$ is given by

$$\sigma_{ind} = \sigma_b = \boldsymbol{P} \cdot \boldsymbol{n} = P$$

where $\boldsymbol{n}$ is the vector normal to the boundary. The total electric field E is obtained as

$$E = \frac{\sigma_f}{\varepsilon_0} - \frac{\sigma_{ind}}{\varepsilon_0} = \frac{1}{\kappa} E_f = \frac{\sigma_f}{\kappa \varepsilon_0}$$

using the Gauss's theorem. Note that

$$\frac{E}{E_f} = \frac{1}{\kappa}$$

where

$$E_f = \frac{\sigma_f}{\varepsilon_0}$$

Then we have

$$\sigma_{ind} = \sigma_f (1 - \frac{1}{\kappa})$$

or

$$E_f (1 - \frac{1}{\kappa}) = \frac{\sigma_{ind}}{\varepsilon_0}$$

where κ is dielectric constant of dielectric.

Gauss' law with dielectrics

$$\varepsilon_0 \int \kappa \boldsymbol{E} \cdot d\boldsymbol{a} = q_f$$

Table: Dielectric constants

Dielectric constants of various substances

Substance	Conditions	Dielectric constant (κ)
Air	gas, 0 °C, 1 atm	1.00059
Methane, CH_4	gas, 0 °C, 1 atm	1.00088
Hydrogen chloride, HCl	gas, 0 °C, 1 atm	1.0046
Water, H_2O	gas, 110 °C, 1 atm	1.0126
	liquid, 20 °C	80.4
Benzene, C_6H_6	liquid, 20 °C	2.28
Methanol, CH_3OH	liquid, 20 °C	33.6
Ammonia, NH_3	liquid, −34 °C	22.6
Mineral oil	liquid, 20 °C	2.24
Sodium chloride, NaCl	solid, 20 °C	6.12
Sulfur, S	solid, 20 °C	4.0
Silicon, Si	solid, 20 °C	11.7
Polyethylene	solid, 20 °C	2.25–2.3
Porcelain	solid, 20 °C	6.0-8.0
Paraffin wax	solid, 20 °C	2.1–2.5
Pyrex glass 7070	solid, 20 °C	4.00

κ(vacuum) = 1.000000
κ(paper) = 3.5
κ(transformer oil) = 4.5

$\kappa(SrTiO_3) = 310$
κ(liquid water at 25°C) = 78.5

The polarization vector is defined as

$$\boldsymbol{P}\cdot\boldsymbol{n} = P = \sigma_{ind} = \varepsilon_0\chi E$$

or

$$\frac{\sigma_{ind}}{\varepsilon_0} = \frac{P}{\varepsilon_0} = \chi E$$

or

$$P = \varepsilon_0\chi E$$

Thus we have

$$E = \frac{\sigma_f}{\varepsilon_0} - \chi E$$

or

$$(1+\chi)E = \frac{\sigma_f}{\varepsilon_0}$$

or

$$E = \frac{1}{1+\chi}\frac{\sigma_f}{\varepsilon_0} = \frac{1}{1+\chi}E_f = \frac{1}{\kappa}E_f$$

leading to the relation

$$\kappa = 1+\chi$$

where χ is called the *electric susceptibility*.

17. Capacitance of dielectric (II)

The capacitance C of the dielectric is defined by

$$C = \frac{Q_f}{V}.$$

where C_0 is the capacitance of the vacuum. The validity of this definition is explained in Sec.

$$C_0 = \frac{Q_f}{V_f} = \frac{Q_f}{E_f d} = \frac{A\sigma_f}{\frac{\sigma_f}{\varepsilon_0} d} = \varepsilon_0 \frac{A}{d}$$

and

$$V = Ed = \frac{1}{\kappa} E_f d = \frac{1}{\kappa} V_f$$

Thus we have the capacitance,

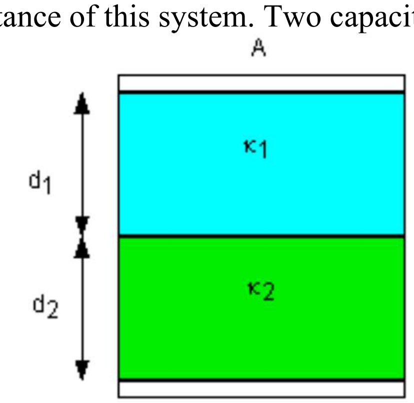
$$C = \frac{Q_f}{V} = \frac{Q_f}{V_f}\frac{V_f}{V} = \kappa C_0 = \kappa \varepsilon_0 \frac{A}{d}$$

or

$$\frac{C}{C_0} = \kappa$$

18. Capacitors with dielectrics in series and in parallel connections

We calculate the capacitance of this system. Two capacitors are connected in series.

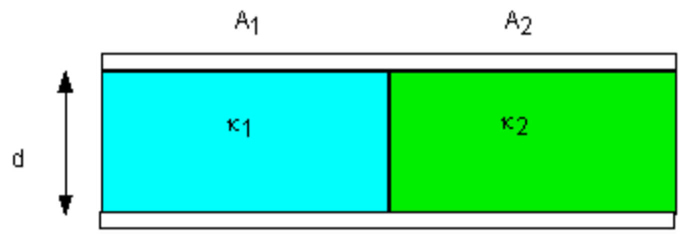


$$C_1 = \varepsilon_0 \kappa_1 \frac{A}{d_1}$$

$$C_2 = \varepsilon_0 \kappa_2 \frac{A}{d_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 \kappa_1 \frac{A}{d_1} \varepsilon_0 \kappa_2 \frac{A}{d_2}}{\varepsilon_0 \kappa_1 \frac{A}{d_1} + \varepsilon_0 \kappa_2 \frac{A}{d_2}} = \frac{\varepsilon_0 A \frac{\kappa_1}{d_1} \frac{\kappa_2}{d_2}}{\frac{\kappa_1}{d_1} + \frac{\kappa_2}{d_2}}$$

Next we calculate the capacitance of the system where two capacitors are connected in parallel



$$C_1 = \varepsilon_0 \kappa_1 \frac{A_1}{d}$$

$$C_2 = \varepsilon_0 \kappa_2 \frac{A_2}{d}$$

$$C = C_1 + C_2 = \varepsilon_0 \kappa_1 \frac{A_1}{d} + \varepsilon_0 \kappa_2 \frac{A_2}{d}$$

$$= \frac{\varepsilon_0}{d}(\kappa_1 A_1 + \kappa_2 A_2)$$

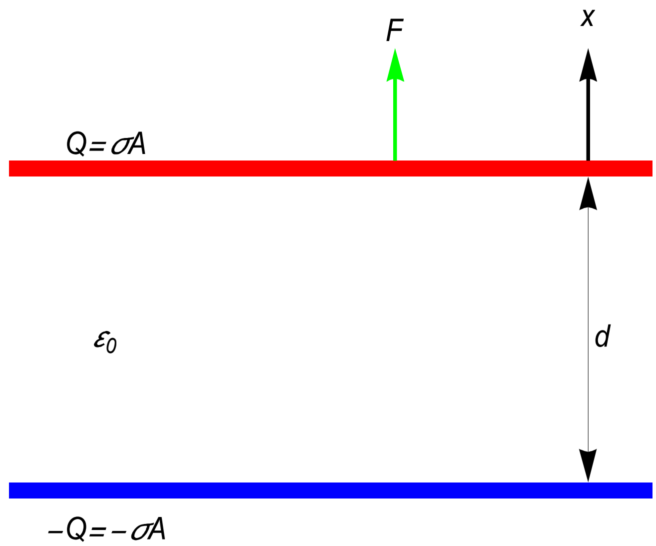
19. Work-energy theorem for capacitance (I)
Walter Lewin: 8.02X Electricity and Magnetism

We consider the capacitance consisting of two conducting plates which are parallel to each other. The separation distance between two plates id *d*. The upper plate is positively charged; $Q = \sigma A$, while the lower late is negatively charged as $-Q = -\sigma A$. The electric field is constant is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

We now consider a case when the upper plate is moved upward by a force *F* (along the *x* direction). Note that the weight of the upper plate is negligibly small. We use the work-energy theorem,

$$\Delta K = W = -\Delta U$$



The work is given by

$$W = Fdx = -\Delta U$$

where F is the conservative force, and U is the potential energy

$$U = \frac{1}{2}QV = \frac{1}{2}(\sigma A)\frac{\sigma}{\varepsilon_0}d = \frac{\sigma^2 Ad}{2\varepsilon_0}$$

$$Q = \sigma A, \qquad V = Ed = \frac{\sigma}{\varepsilon_0}d$$

Since

$$\Delta U = \frac{\sigma^2 A}{2\varepsilon_0}\Delta x$$

the force F is

$$F_x = -\frac{dU}{dx} = -\frac{\sigma^2 A}{2\varepsilon_0} \ (<0)$$

which is an attractive force. If you want to move the plate to the upward, you need to apply an external force

$$F_{ext} = \frac{\sigma^2 A}{2\varepsilon_0}$$

So we have the work

$$W = F_{ext}x = \frac{\sigma^2 Ax}{2\varepsilon_0} = \frac{\sigma^2 Ax}{2\varepsilon_0}$$

where Ax is the volume. The electric field energy density is obtained as

$$\boxed{\frac{W}{Ax} = \frac{\sigma^2}{2\varepsilon_0} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2}$$

20. Work energy theorem for capacitance

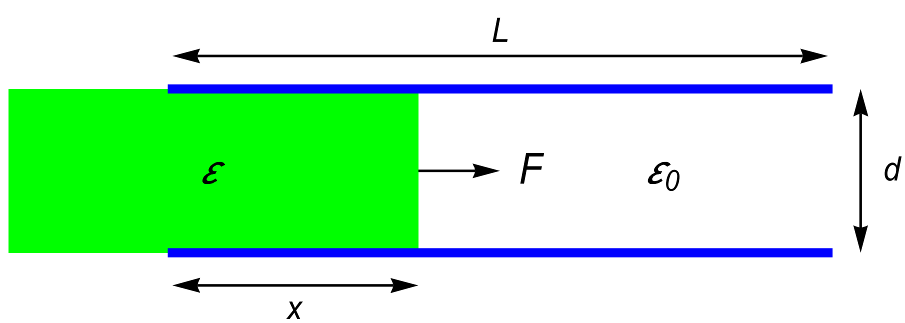
(1) Force on a capacitance plate: (Problem 3-26) Purcell and Morin

A parallel-plate capacitor consists of a fixed plate and a moval plate that is allowed to slide in the direction parallel to the plates. Let x be the distance of overlap as shown in Fig. The separation between the plates is fixed.

(a) Assume that the plates are electrically isolated, so that their charges $\pm Q$ are constant. In terms of Q and the (variable) capacitance C, derive an expression for the leftward force on the movable plate.

(b) Now assume that the plates are connected to a battery, so that the potential difference V is held constant. In terms of V and the capacitance C, derive an expression for the force.

(c) If the movable plate is held in place by an opponent force, then either of the above two setups could be the relevant one, because nothing is moving. So the forces in (a) and (b) should be equal. Verify that this is the case.



(a) Q = constant

Work-energy theorem

$$\Delta K = \Delta W = -\Delta U$$

$$\Delta W = \boldsymbol{F} \cdot d\boldsymbol{r} = -\Delta U$$

where

$$U = \frac{1}{2}QV = \frac{Q^2}{2C}$$

with $Q = CV$. The force $\boldsymbol{F}$ is given by

$$\boldsymbol{F} = -\nabla U$$

or

$$F_x = -\frac{dU}{dx} = -\frac{Q^2}{2}\frac{d}{dx}\frac{1}{C} = \frac{Q^2}{2C^2}\frac{dC}{dx} \qquad (F_x > 0).$$

for the 1D system. Note that

$$C = \varepsilon\frac{Lx}{d} + \varepsilon_0\frac{L(L-x)}{d} = \frac{L}{d}[\varepsilon x + \varepsilon_0(L-x)]$$

The capacitance C increases with increasing x.

$$\frac{dC}{dx} = \frac{L}{d}(\varepsilon - \varepsilon_0) \qquad (>0)$$

(b) V = constant

Work-energy theorem:

$$\Delta K = \Delta W = \Delta W_b - \Delta U = \boldsymbol{F}\cdot\Delta\boldsymbol{r}$$

where W_b is the work required to move each of the charge increment.

$$\Delta W_b = V\Delta Q, \qquad \Delta U = \frac{1}{2}V\Delta Q$$

or

$$\Delta W_b = 2\Delta U$$

Then we have

$$\Delta W = \Delta W_b - \Delta U = 2\Delta U - \Delta U = \Delta U = \boldsymbol{F}\cdot\Delta\boldsymbol{r}$$

or

$$F_x = \frac{dU}{dx} = \frac{1}{2}V^2\frac{dC}{dx} \qquad (F_x > 0).$$

for the 1D system, where

$$\frac{dC}{dx} = \frac{L}{d}(\varepsilon - \varepsilon_0) \qquad (>0)$$

21. Displacement vector *D*

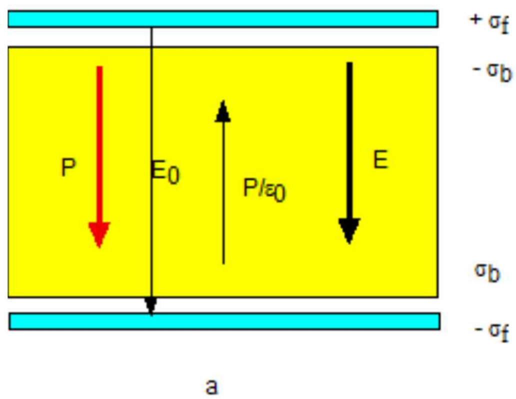


Fig. *P* is the polarization of the dielectric. σ_f is the free surface charge density due to the free charges located on the two parallel plates. $\sigma_b = \sigma_{ind}$, σ_b is the bound surface charge density due to the polarization of the dielectric. E_0 is an external electric field. *E* is an electric field inside the dielectrics. σ_b is equal to *P*. $E = E_0 - \frac{P}{\varepsilon_0}$. *P* is related to *E* through $P = \varepsilon_0 \chi E$.

The external field $\boldsymbol{E}_0$ inside the air (the space between two parallel metal plates is air) is given by

$$E_0 = E_f = \frac{\sigma_f}{\varepsilon_0}$$

or

$$\varepsilon_0 \oint \boldsymbol{E}_f \cdot d\boldsymbol{a} = q_f$$

The electric field inside the dielectric (the space between two parallel metal plates is filled with dielectric) is given by

$$E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$$

or

$$\varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{a} = q_{eff} = q_f - q_b$$

where q_f is the free charge density and q_b is the bound charge density.

Here we define the electric displacement $\boldsymbol{D}$ by

$$D = \sigma_f = \varepsilon_0 E_f \qquad \text{(electric displacement)}$$

or

$$\oint \boldsymbol{D} \cdot d\boldsymbol{a} = q_f$$

This equation states Gauss' law in its general form. It is applicable to any dielectric medium as well as to a vacuum. This is a useful way to express Gauss' law, in the context of dielectrics, because it makes reference only to free charges, and free charge is the stuff we control (Griffiths, Introduction to electrodynamics).

Since $\boldsymbol{E}_f = \kappa \boldsymbol{E}$, $\boldsymbol{D}$ is described as

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}_f = \varepsilon_0 \kappa \boldsymbol{E}$$

Then we get

$$q_f = \varepsilon_0 \oint \boldsymbol{E}_f \cdot d\boldsymbol{a} = \varepsilon_0 \kappa \oint \boldsymbol{E} \cdot d\boldsymbol{a} = \oint \boldsymbol{D} \cdot d\boldsymbol{a}$$

or

$$\varepsilon_0 \kappa \oint \boldsymbol{E} \cdot d\boldsymbol{a} = q_f \qquad \text{(Gauss' law with dielectric)}$$

or

$$\int (\nabla \cdot \kappa \boldsymbol{E}) dV = \oint \boldsymbol{E}_f \cdot d\boldsymbol{a} = \frac{1}{\varepsilon_0} Q_f = \frac{1}{\varepsilon_0} \int \rho_f dV$$

leading to the formula

$$\nabla \cdot (\kappa \boldsymbol{E}) = \frac{1}{\varepsilon_0}\rho_f$$

22. Application of the Gauss' law

We apply the Gauss theorem on the Gaussian surface (cylindrical surface)

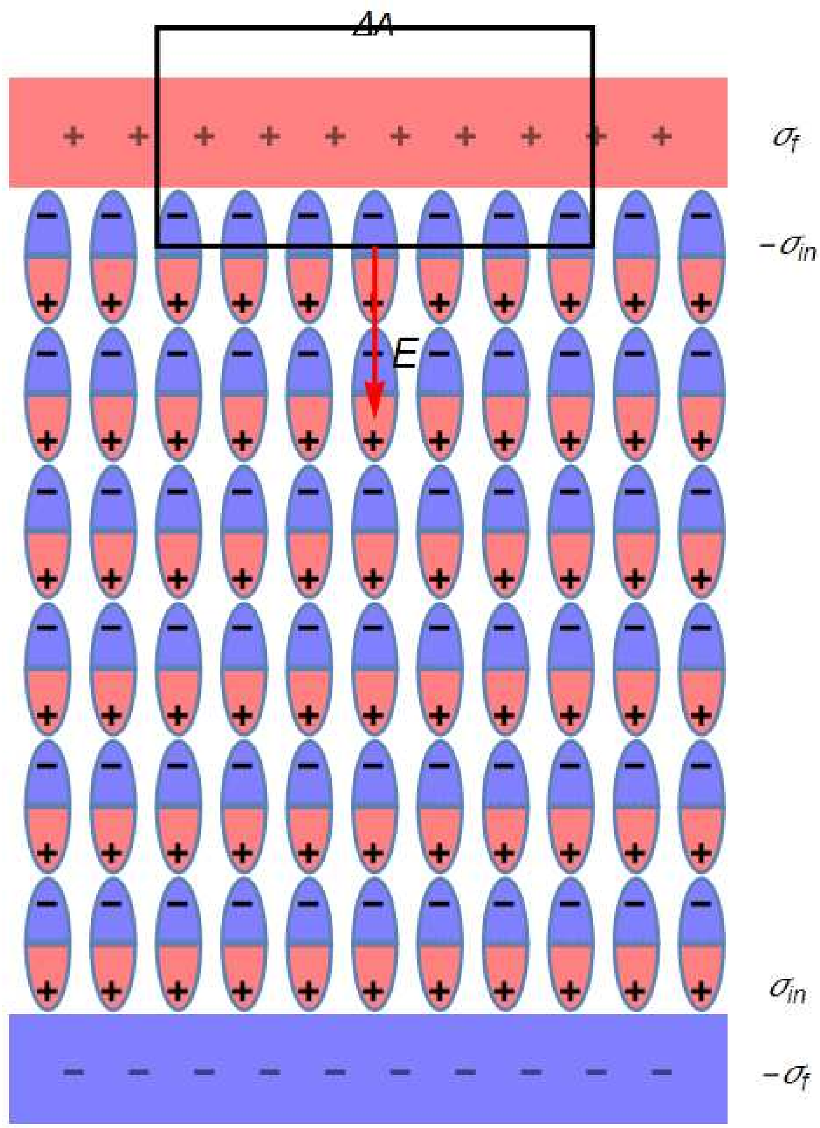
$$E\Delta A = \frac{1}{\varepsilon_0}(\sigma_f - \sigma_{in})\Delta A$$

or

$$E = \frac{1}{\varepsilon_0}(\sigma_f - \sigma_{in})$$

Since $\sigma_f = D$ and $\sigma_{in} = P$, we have

$$\varepsilon_0 E = D - P, \qquad D = \varepsilon_0 E + P$$



23. Example: *D* and *E* for the capacitor

We consider the simple case of the capacitor where the dielectric with *k* between two parallel plates.

The displacement vector *D* is given by

$$D = \sigma_f$$

D is related to the electric field *E* by

$$D = \varepsilon_0 \kappa E$$

or

$$E = \frac{D}{\varepsilon_0 \kappa} = \frac{\sigma_f}{\varepsilon_0 \kappa}$$

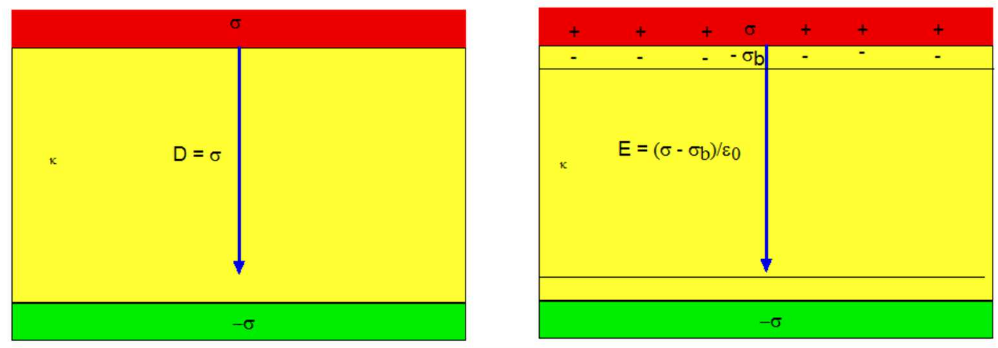


Fig. $\sigma = \sigma_f$ and $\sigma_{ind} = \sigma_b$ in this Fig.

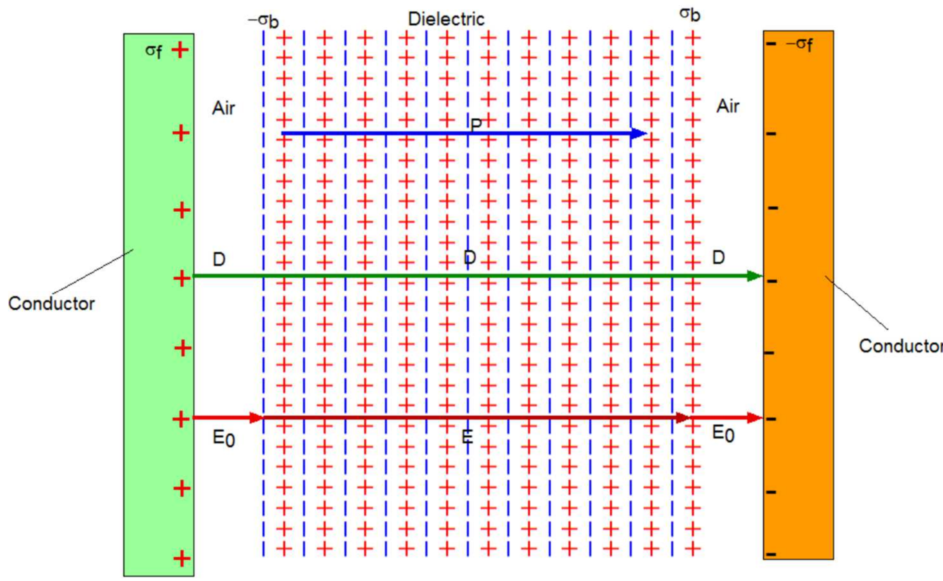
The electric field E is also derived as

$$E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$$

The bound surface charge σ_b is obtained as

$$\sigma_b = \sigma_f - \varepsilon_0 E = \sigma_f - \varepsilon_0 \frac{\sigma_f}{\varepsilon_0 \kappa} = \sigma_f (1 - \frac{1}{\kappa})$$

In summary, we show the schematic diagram for the fields ***D*** and ***E*** in the dielectric in the parallel-plate capacitor; the displacement vector ***D*** depends only on the free charge and is the same inside and outside (air gaps).



$$D = \sigma_f \qquad E_0 = \frac{\sigma_f}{\varepsilon_0}$$

$$D = \sigma_f \qquad P = \sigma_b \qquad E = \frac{\sigma_f - \sigma_b}{\varepsilon_0} = \frac{\sigma_f}{\varepsilon_0 \kappa}$$

$$D = \sigma_f \qquad E_0 = \frac{\sigma_f}{\varepsilon_0}$$

24. Maxwell/s equation with *E*, *D*, and *P*

The effect of the polarization is equivalent to a charge density ρ_b given by

$$\rho_b = -\nabla \cdot \boldsymbol{P}$$

The divergence of ***E*** is related to the effective charge density ρ_{eff} by

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_{eff}}{\varepsilon_0} = \frac{\rho_f + \rho_b}{\varepsilon_0} = \frac{\rho_f}{\varepsilon_0} - \frac{1}{\varepsilon_0} \nabla \cdot \boldsymbol{P}$$

where ρ_f is a free charge density. This equation is rewritten as

$$\nabla \cdot (\varepsilon_0 \boldsymbol{E} + \boldsymbol{P}) = \rho_f$$

We define ***D*** as

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

with

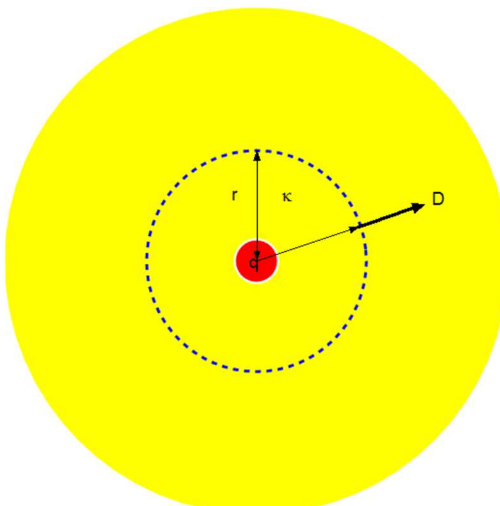
$$\nabla \cdot \boldsymbol{D} = \rho_f$$

In summary

$$\boldsymbol{D} = \varepsilon_0 \kappa \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}, \qquad \nabla \cdot \boldsymbol{D} = q_f$$

25. Example: *D* and *E* for the simple case with spherical symmetry

We consider the simple case of dielectric sphere where the point charge is located at the center.



We apply the Gauss' law for the Gaussian surface (dashed line)

$$\int \boldsymbol{D} \cdot d\boldsymbol{a} = q_f = q$$

or

$$D(4\pi r^2) = q_f$$

or

$$D = \frac{q_f}{4\pi r^2}$$

where q_f (= q) is the free charges. The electric field $\boldsymbol{E}$ is related to $\boldsymbol{D}$ by a relation

$$D = \varepsilon_0 \kappa E$$

Then we have

$$E = \frac{D}{\varepsilon_0 \kappa} = \frac{q}{4\pi\varepsilon_0 \kappa r^2}$$

The effective charge inside the dashed line (q_{eff}) is evaluated as

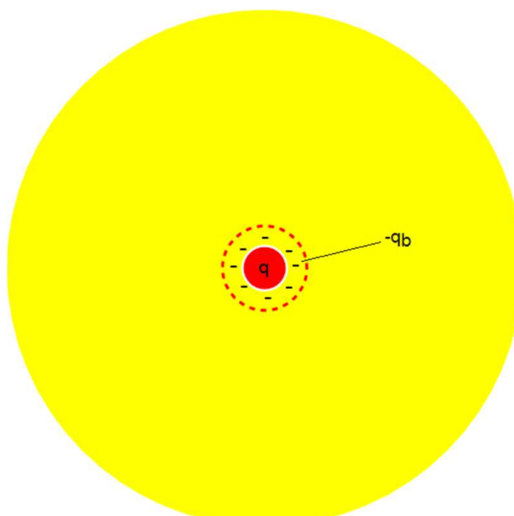
$$q_{eff} = \varepsilon_0 \int \boldsymbol{E} \cdot d\boldsymbol{a} = \varepsilon_0 \frac{q}{4\pi\varepsilon_0 \kappa r^2}(4\pi r^2) = \frac{q}{\kappa}$$

Here q_{eff} consists of free charge (q) and bound charge (q_{b}).

$$q_{eff} = q - q_b = \frac{q}{\kappa}$$

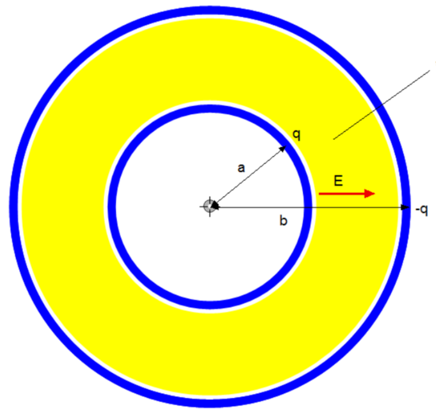
or

$$q_b = q(1 - \frac{1}{\kappa})$$



26. Example Problem 25-53 (SP25-53)

The space between two concentric spherical shells of radii b = 1.70 cm and a = 1.20 cm is filled with a substance of dielectric constant κ = 23.5. A potential difference V = 73.0 V is applied across the inner and outer shells. Determine (a) the capacitance of the device, (b) the free charge q on the inner shell, and (c) the charge q’ induced along the surface of the inner shell.



((**Solution**))
a = 1.20 cm
b = 1.70 cm
κ = 23.5
V_{ba} = 73.0 V

We apply the Gauss' law for the Gaussian surface (dashed line)

$$\int \boldsymbol{D}\cdot d\boldsymbol{a} = q_f = q \qquad \text{(true charge)}$$

Then we have

$$D(4\pi r^2) = q$$

or

$$D = \frac{q}{4\pi r^2}$$

Using the relation give by

$$D = \varepsilon_0 \kappa E$$

The electric field E is derived as

$$E = -\frac{dV}{dr} = \frac{D}{\varepsilon_0 \kappa} = \frac{q}{4\pi\varepsilon_0 \kappa r^2}$$

Then we have

$$V_{ab} = -\int_a^b \frac{q}{4\pi\varepsilon_0 \kappa r^2} dr = \frac{q}{4\pi\varepsilon_0 \kappa}\left(\frac{1}{b} - \frac{1}{a}\right) = -V_{ba}$$

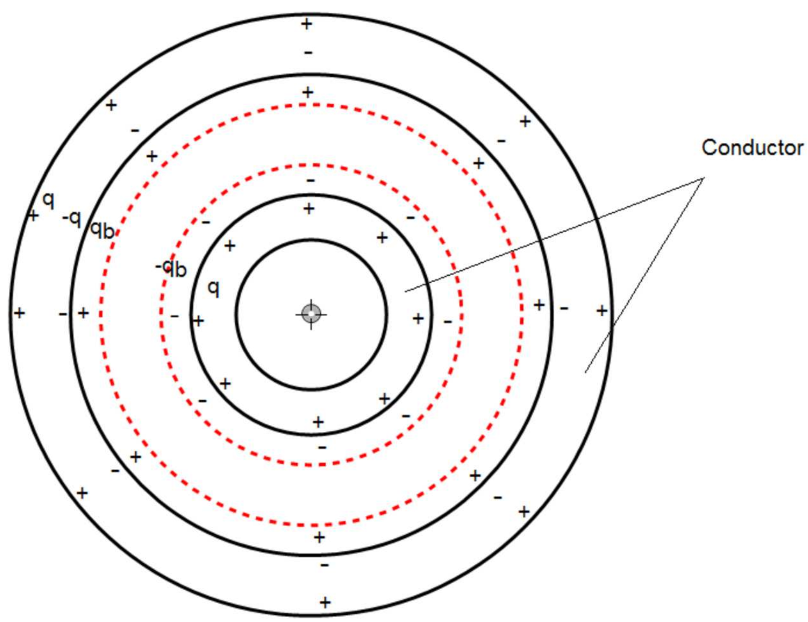
(a)

$$Q_a = q = CV_{ba}$$

$$C = \frac{q}{V_{ba}} = \frac{4\pi\varepsilon_0\kappa}{\frac{1}{a} - \frac{1}{b}} = 0.1067nF$$

(b)

$$q = CV_{ba} = 0.1067nF \times 73V = 7.79nC$$



(c)
Gaussian surface (dotted line in the vicinity of $r = a$)

$$\varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{a} = q_{eff} = q - q_b$$

where q_b is the bound charge (induced charge)

For the Gaussian surface just outside $r = a$,

$$E(4\pi a^2) = \frac{q - q_b}{\varepsilon_0} \qquad \text{or} \qquad E = \frac{q - q_b}{4\pi\varepsilon_0 a^2}$$

The electric field E is also given by

$$E = \frac{q}{4\pi\varepsilon_0 \kappa a^2}$$

Using the dielectric constant κ, we have

$$q - q_b = \varepsilon_0 E(4\pi a^2) = \varepsilon_0 \frac{q}{4\pi\varepsilon_0 \kappa a^2}(4\pi a^2) = \frac{q}{\kappa}$$

or

$$q_b = q(1 - \frac{1}{\kappa}) = 7.46 \text{ nC}$$

REFERENCES

J. Reitz, F.J. Milford, and R.W. Christy, *Foundations of Electromagnetic Theory*, 3rd edition (Addison-Wesley, 1980).
E.M. Purcell and D.J. Morin, *Electricity and Magnetism*, 3rd edition (Cambridge, 2013).
R.P. Feynman, R.B. Leighton, and M. Sands, Lectures on Physics II (Basic Book, 2010).

APPENDIX

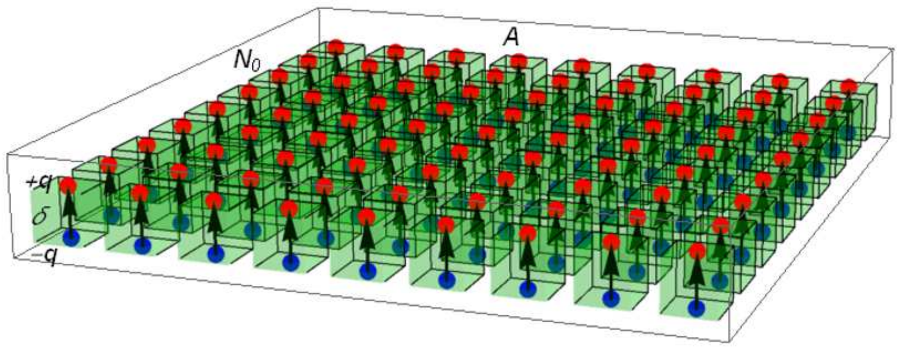
Surface charge density in terms of the polarization vector ((Feynman))

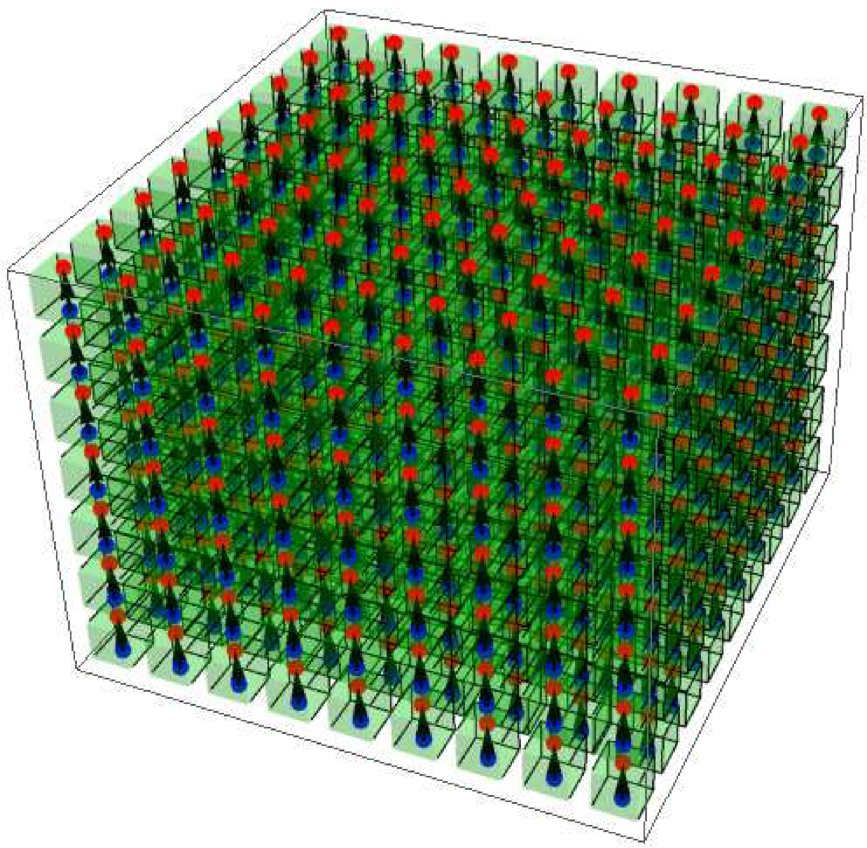
We now consider the situation in which the polarization vector $\boldsymbol{P}$ is not everywhere the same. If the polarization is not constant, we would expect in general to find a charge density in the volume, because more charge might come into one side of a small volume element than leaves it on the other. How can we find out how much charge is gained or lost from a small volume?

We calculate how much charge moves across any imaginary surface when the material is polarized. The amount of charge that goes across a surface is just P times the surface area if the polarization is *normal* to the surface. Of course, if the polarization is *tangential* to the surface, no charge moves across it. Following the same arguments, it is easy to see that the charge moved across any surface element is proportional to the *component* of $\boldsymbol{P}$ *perpendicular* to the surface.

(a) The case of polarization vector which is normal to the top of the surface

We assume that the electric dipole moment is normal to the top of the surface.





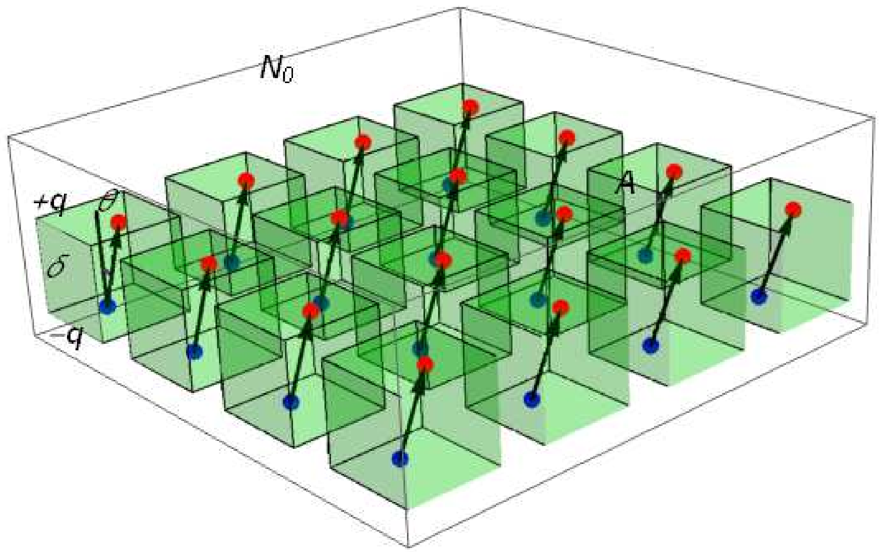
The surface charge density is obtained as

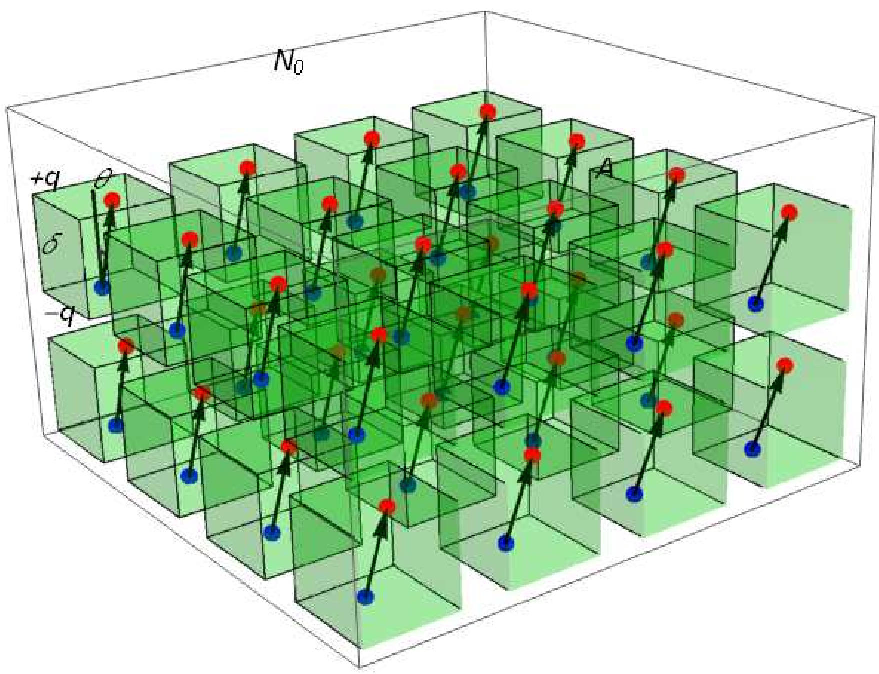
$$\sigma_P = \frac{N_0 q}{A} = \frac{N_0(q\delta)}{A\delta} = \frac{N_0 p}{V} = P$$

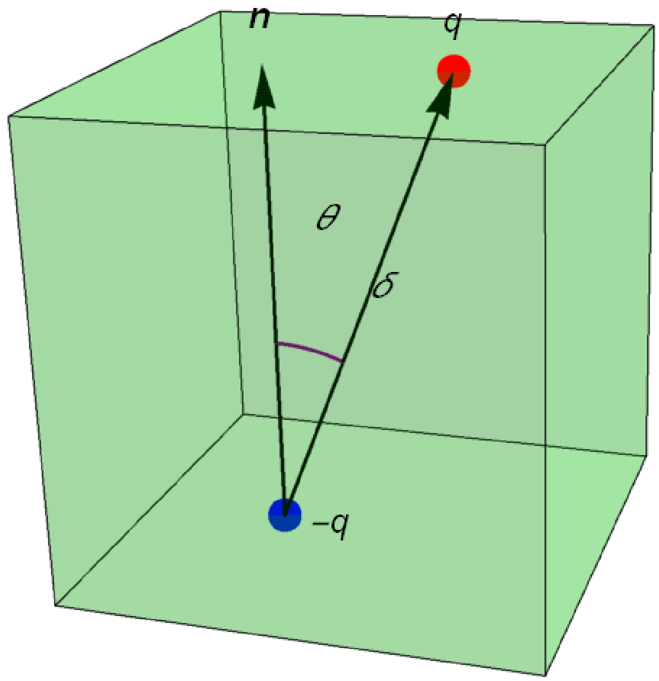
which is equal to the magnitude of the polarization vector, where $V = A\delta$

(b) The case of polarization vector which is not normal to the top of the surface

We assume that the electric dipole moment is not normal to the top of the surface.


N_0
$+q$
θ
δ
$-q$
A


N_0
$+q$
θ
δ
$-q$
A



The surface charge density is

$$\sigma_P = \frac{N_0 q}{A} = \frac{N_0 q\delta\cos\theta}{A\delta\cos\theta} = \frac{N_0 p\cos\theta}{V} = P\cos\theta\,,$$

or

$$\sigma_P = \boldsymbol{P}\cdot\boldsymbol{n}\,.$$

where $\qquad V = A\delta\cos\theta\,.$

((**Feynman**))

In his book, Feynman derived the expression $\sigma_P = \boldsymbol{P}\cdot\boldsymbol{n}$ using the following Fig.

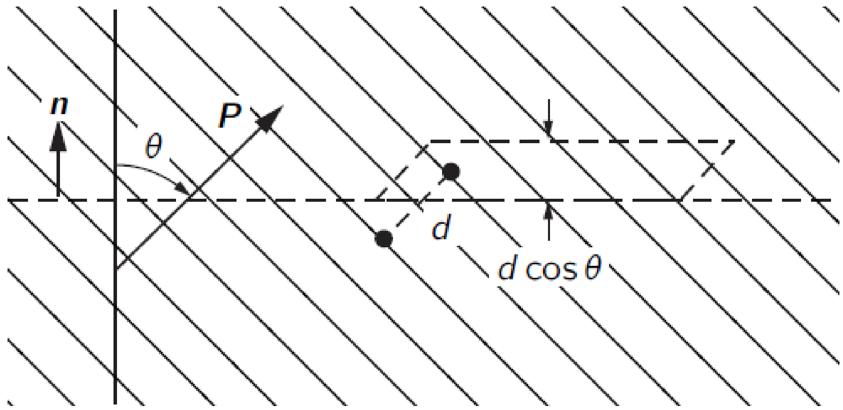
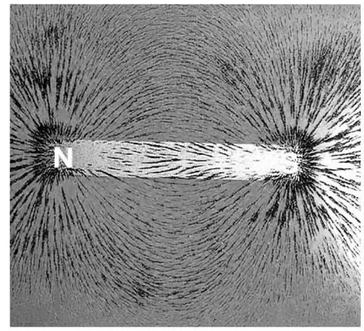


Fig. The charge moved across an element of an imaginary surface in a dielectric is proportional to the component of ***P*** normal to the surface. $d = \delta$.

REFERENCES

R.P. Feynman, R.B. Leighton, and M. Sands, The Feynman Lectures on Physics Vol.2 (Basic Book, 2010).

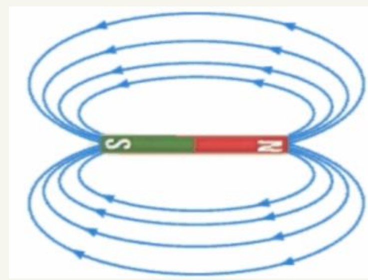
The magnetic field



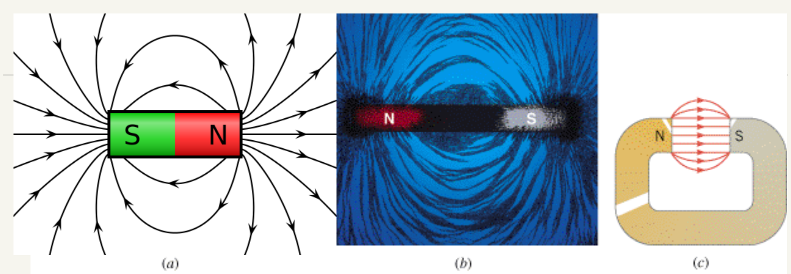
A bar magnet has a magnetic field around it. This field is 3D in nature and often represented by lines LEAVING north and ENTERING south

The magnetic field is a vector that has both magnitude and direction.

The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.

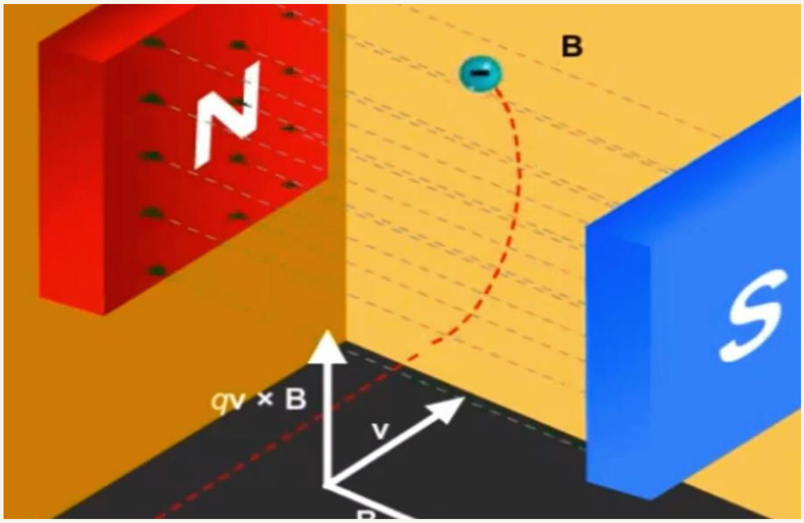


The properties of magnetic field line



1. The lines originate from the north pole and end on the south pole; they do not start or stop in mid-space.
2. The magnetic field at any point is tangent to the magnetic field line at that point.
3. The strength of the field is proportional to the number of lines per unit area that passes through a surface oriented perpendicular to the lines.
4. The magnetic field lines will never come to cross each other.

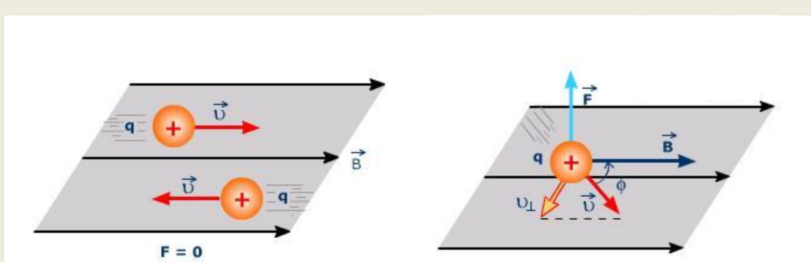
Magnetic force on moving charge



Magnetic force on moving charge

When a charge is placed in a magnetic field, it experiences a magnetic force if two conditions are met:

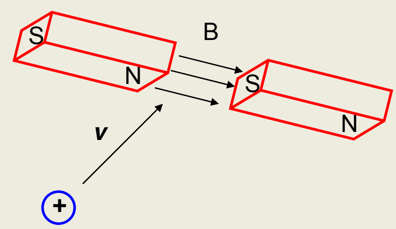
1. The charge must be moving. No magnetic force acts on a stationary charge.
2. The velocity of the moving charge must have a component that is perpendicular to the direction of the field.



Properties of the magnetic force on a charged particle moving in a magnetic field

We can define a magnetic field **B** at some point in space in terms of the magnetic force F_B the field exerts on a charged particle moving with a velocity **v**, which we call the test object.

Experiments on various charged particles moving in a magnetic field give the following results:

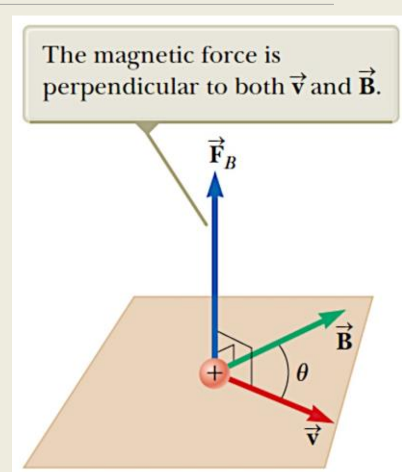


(1) The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.

Properties of the magnetic force on a charged particle moving in a magnetic field

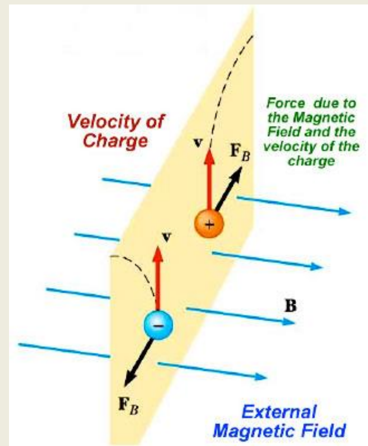
(2) When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.

(3) When the particle's velocity vector makes any angle θ ≠ 0 with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and ***B***; that is, F_B is perpendicular to the plane formed by **v** and **B.**



Properties of the magnetic force on a charged particle moving in a magnetic field

(4) The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.



Properties of the magnetic force on a charged particle moving in a magnetic field

(5) The magnitude of the magnetic force exerted on the moving particle is proportional to sinθ, where θ is the angle the particle's velocity vector makes with the direction of **B**.

$$F_B = qvB\sin$$

Vector expression for the magnetic force on a charged particle moving in a magnetic field

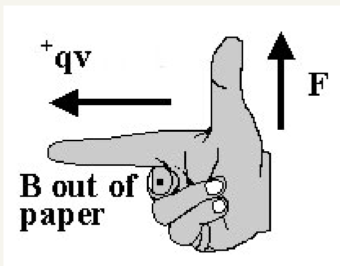
Magnetic Force — Magnetic Field

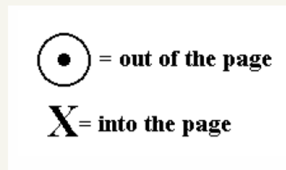
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

velocity of charge

Direction of the magnetic force?
Right Hand Rule

To determine the DIRECTION of the force on a **POSITIVE** charge we use a special technique that helps us understand the 3D perpendicular nature of magnetic fields.





Unit of Magnetic Field

SI unit of magnetic field is the newton per coulomb-meter per second, which is called the tesla (T):

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C.m/s}}$$

Because a coulomb per second is defined to be an ampere,

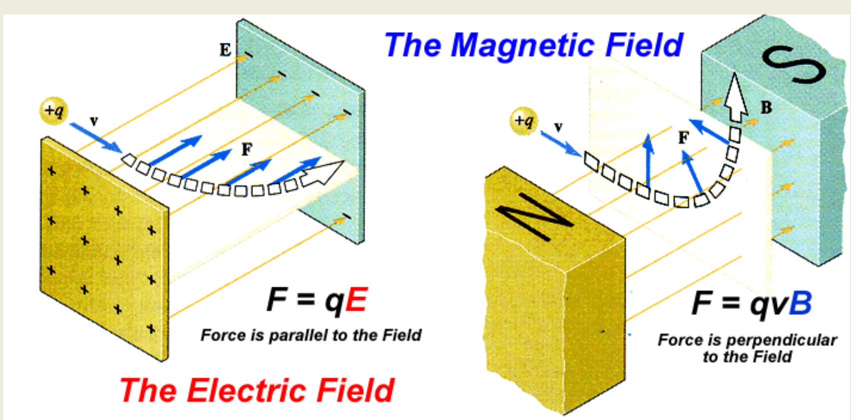
$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A.m}}$$

A non-SI magnetic-field unit in common use, called the gauss (G), is related to the tesla through the conversion 1 T = 10^4 G.

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	**30**
Strong conventional laboratory magnet	**2**
Medical MRI unit	**1.5**
Magnetic Bar	**10^{-2}**
Surface of the Sun	**10^{-2}**
Surface of the Earth	**0.5×10^{-4}**
Inside human brain due to nerve impulses	**10^{-13}**

Motion of charge particle in
- Electric field
- Magnetic field



Differences between Electric and Magnetic Forces

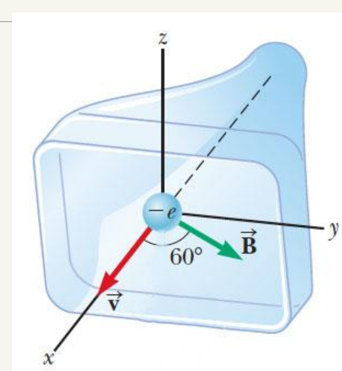
1. The electric force vector is along the direction of the electric field, whereas the magnetic force vector is perpendicular to the magnetic field.
2. The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
3. The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application.

The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. The field can alter the direction of the velocity vector, but it cannot change the speed or kinetic energy of the particle.

Example 1

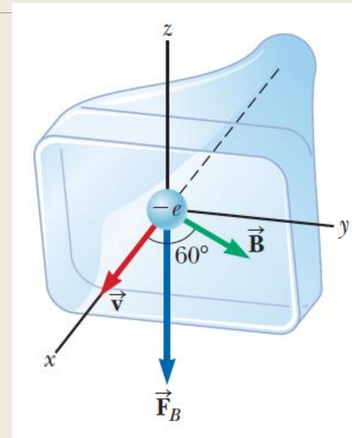
An electron in an old-style television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the x axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the x axis and lying in the xy plane.

Calculate the magnetic force on the electron.



Solution

Use one of the right-hand rules to determine the direction of the force on the electron



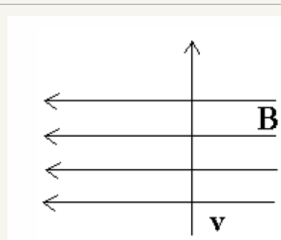
$$F_B = qvB\sin$$

= **(1.6 × 10^{-19} C)(8.0 × 10^{6} m/s)(0.025 T)(sin 60^{o})**

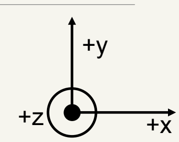
= 2.8 × 10^{-14} N

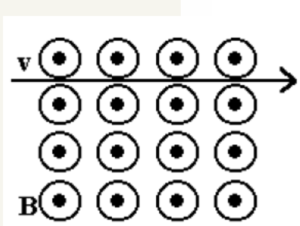
Example 2

Determine the direction of the unknown variable for a proton moving in the field using the coordinate axis given

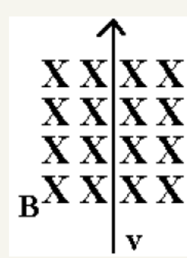


B = -x
v = +y
F = **+z**





B =+Z
v = +x
F = **-y**



B = -z
v = +y
F = **-x**

Example 3

A spatially uniform magnetic field cannot exert a magnetic force on a particle in which of the following circumstances? There may be more than one correct statement.

- (a) The particle is charged.
- (b) The particle moves perpendicular to the magnetic field.
- (c) The particle moves parallel to the magnetic field.
- (d) The magnitude of the magnetic field changes with time.
- (e) The particle is at rest.

Example 4

A particle with electric charge is fired into a region of space where the electric field is zero. It moves in a straight line. Can you conclude that the magnetic field in that region is zero?

- (a) Yes, you can.
- (b) No; the field might be perpendicular to the particle's velocity.
- (c) No; the field might be parallel to the particle's velocity.
- (d) No; the particle might need to have charge of the opposite sign
- to have a force exerted on it.
- (e) No; an observation of an object with electric charge gives no information about a magnetic field.

Example 5

Classify each of the following statements as a characteristic (a) of electric forces only, (b) of magnetic forces only, (c) of both electric and magnetic forces, or (d) of neither electric nor magnetic forces.

(1) The force is proportional to the magnitude of the field exerting it.

(2) The force is proportional to the magnitude of the charge of the object on which the force is exerted.

(3) The force exerted on a negatively charged object is opposite in direction to the force on a positive charge.

(4) The force exerted on a stationary charged object is nonzero.

(5) The force exerted on a moving charged object is zero.

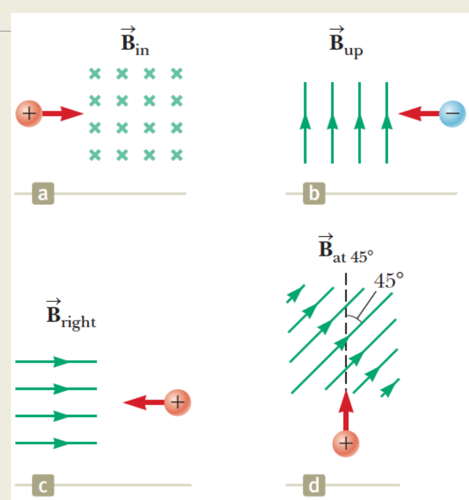
(6) The force exerted on a charged object is proportional to its speed.

(7) The force exerted on a charged object cannot alter the object's speed.

(8) The magnitude of the force depends on the charged object's direction of motion.

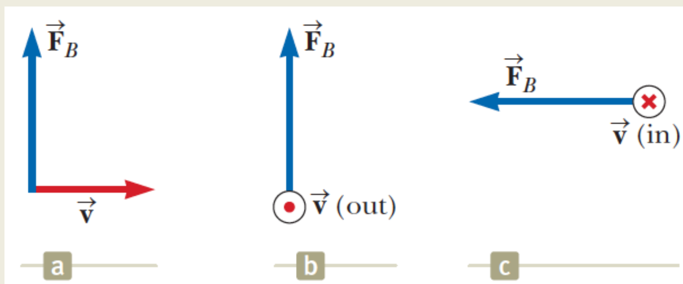
Example 6

Determine the initial direction of the deflection of charged particles as they enter the magnetic fields shown in the Figure



Example 7

Find the direction of the magnetic field acting on a positively charged particle moving in the various situations shown in the Figure if the direction of the magnetic force acting on it is as indicated.



Solve by Your self

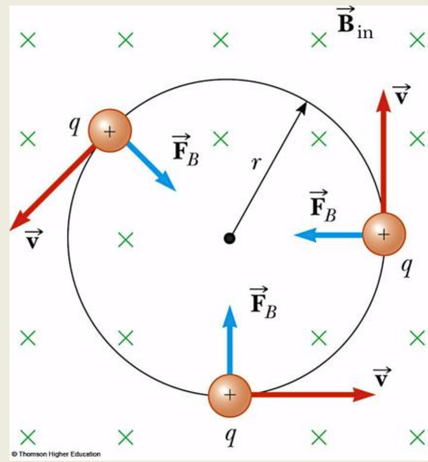
- Two charged particles are projected in the same direction into a magnetic field perpendicular to their velocities. If the particles are deflected in opposite directions, what can you say about them?
- How can the motion of a moving charged particle be used to distinguish between a magnetic field and an electric field?
- Can a constant magnetic field set into motion an electron initially at rest? Explain your answer.

Charged Particle in a Magnetic Field

Consider a +ve charged particle moving in an external magnetic field with its velocity perpendicular to the field.

The magnetic force is always directed toward the center of the circular path.

The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle.



We use the particle under a net force model to write Newton's second law for the particle:

$$F = F_B = ma$$

Because the particle moves in a circle, we also model it as a particle in uniform circular motion and we replace the acceleration with centripetal acceleration:

$$F_B = qvB = \frac{mv^2}{r}$$

This expression leads to the following equation for the radius of the circular path:

$$r = \frac{mv}{qB}$$ **Radius of the circular path**

The **radius** of the path is proportional to the linear momentum *mv* of the particle and inversely proportional to the magnitude of the charge *q* on the particle and to the magnitude of the magnetic field B.

The angular speed of the particle

$$r = \frac{mv}{qB}$$

$$= \frac{v}{r} = \frac{qB}{m}$$ angular speed

The period of the motion (the time interval the particle requires to complete one revolution) is equal to the circumference of the circle divided by the speed of the particle:

$$T = \frac{2\ \ r}{v} = \frac{2}{\ } = \frac{2\ \ m}{qB}$$ period of the motion

These results show that the angular speed of the particle and the period of the circular motion do not depend on the speed of the particle or on the radius of the orbit.

The angular speed ω is often referred to as the cyclotron frequency because charged particles circulate at this angular frequency in the type of accelerator called a cyclotron.

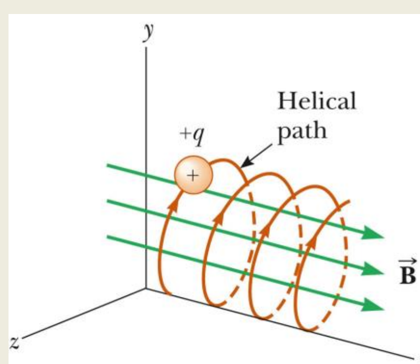
General Case

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to **B**, its path is a helix.

Same equations apply,

with

$$V = \sqrt{V_y^2 + V_z^2}$$



Example 1

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. **Find the speed of the proton.**

Solution

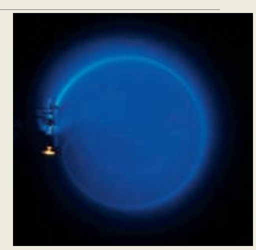
$$= \frac{v}{r} = \frac{qB}{m} \quad \Rightarrow \quad v = \frac{qBr}{m}$$

$$v = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(0.35\ \mathrm{T})(0.14\ \mathrm{m})}{1.67 \times 10^{-27}\ \mathrm{kg}}$$

$$= \boxed{4.7 \times 10^{6}\ \mathrm{m/s}}$$

Example 2

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm.



(A) What is the magnitude of the magnetic field?

(B) What is the angular speed of the electrons?

Solution (A) the magnitude of the magnetic field

$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}m_e v^2 - 0\right) + (q\,\Delta V) = 0$$

$$v = \sqrt{\frac{-2q\,\Delta V}{m_e}}$$

$$v = \sqrt{\frac{-2(-1.60 \times 10^{-19}\ \mathrm{C})(350\ \mathrm{V})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 1.11 \times 10^7\ \mathrm{m/s}$$

$$r = \frac{mv}{qB}$$

$$\boxed{B = \frac{m_e v}{er}}$$

$$B = \frac{m_e v}{er}$$

$$B = \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.11 \times 10^7\ \mathrm{m/s})}{(1.60 \times 10^{-19}\ \mathrm{C})(0.075\ \mathrm{m})} = 8.4 \times 10^{-4}\ \mathrm{T}$$

Solution B the angular speed of the electrons

$$\omega = \frac{v}{r} = \frac{1.11 \times 10^7\ \mathrm{m/s}}{0.075\ \mathrm{m}} = 1.5 \times 10^8\ \mathrm{rad/s}$$

$$\omega = (1.5 \times 10^8\ \mathrm{rad/s})(1\ \mathrm{rev}/2\pi\ \mathrm{rad}) = 2.4 \times 10^7\ \mathrm{rev/s}$$

Applications

- Velocity Selector
- Mass Spectrometer
- The Cyclotron

Applications involving charged particles moving in a magnetic field

In many applications, charged particles will move in the presence of both magnetic and electric fields.

In that case, the total force is the sum of the forces due to the individual fields.

In general (The Lorentz force):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Electric Force **Magnetic Force**

Velocity Selector

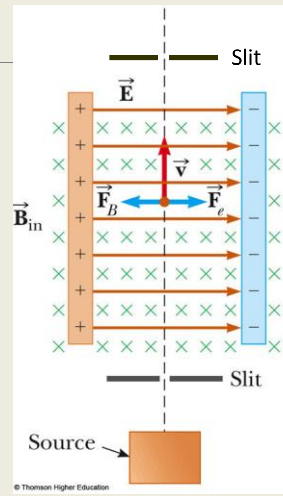
A uniform electric field is perpendicular to a uniform magnetic field.

When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line.

$$qE = qvB$$

This selects particles with velocities of the value

$$v = \frac{E}{B}$$



Mass Spectrometer

A mass spectrometer separates ions according to their mass-to-charge ratio.

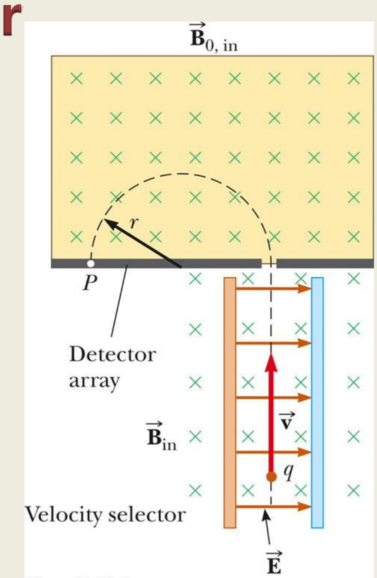
A beam of ions passes through a velocity selector and then enters a second magnetic field where the ions move in a semicircle of radius *r* before striking a detector at P.

From the equation

$$r = \frac{mv}{qB_o}$$

The ratio of m/q

$$\frac{m}{q} = \frac{rB_o}{v}$$



Mass Spectrometer

$$\frac{m}{q} = \frac{rB_o}{v}$$

The velocity is given by the velocity selector of the first part as

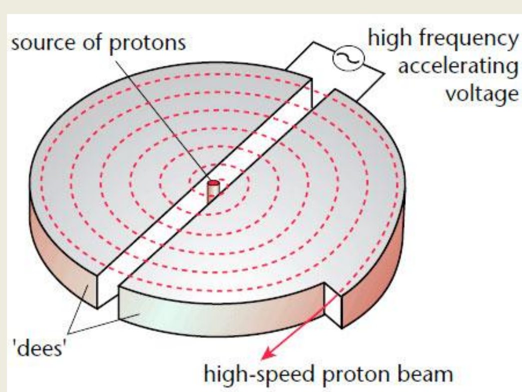
$$v = \frac{E}{B}$$

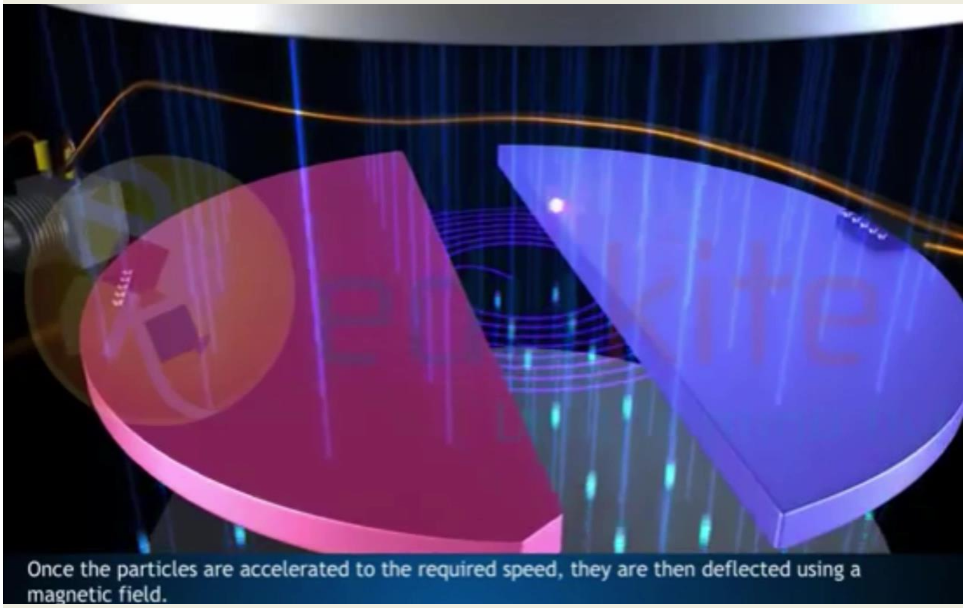
$$\frac{m}{q} = \frac{rB_o B}{E}$$

we can determine **m /q** by measuring the radius of curvature and knowing the field magnitudes **B, B**$_0$, and **E.**

The Cyclotron

A cyclotron is a device that can accelerate charged particles to very high speeds.





The Cyclotron

We can obtain an expression for the kinetic energy of the ion when it exits the cyclotron in terms of the radius R of the dees.

we know that

$$\frac{m}{q} = \frac{RB_o}{v} \quad \Rightarrow \quad v = \frac{qBR}{m}$$

the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

Solve by your self

1. An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2.00 mT. If the speed of the electron is 1.50×10^7 m/s, determine (a) the radius of the circular path and (b) the time interval required to complete one revolution.
2. An electron moves in a circular path perpendicular to a constant magnetic field of magnitude 1.00 mT. The angular momentum of the electron about the center of the circle is 4.00×10^{-25} kg.m^2/s. Determine (a) the radius of the circular path and (b) the speed of the electron.
3. Consider the mass spectrometer. The magnitude of the electric field between the plates of the velocity selector is 2.50×10^3 V/m, and the magnetic field in both the velocity selector and the deflection chamber has a magnitude of 0.035T. Calculate the radius of the path for a singly charged ion having a mass $m = 2.18 \times 10^{-26}$ kg.

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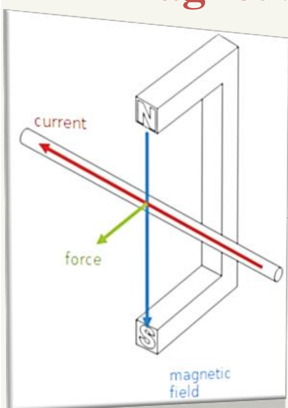
Magnetism and Alternating Current



Unit 1: Magnetic Fields

Lecture 3: Magnetic force acting a current-carrying conductor

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

1.5 Torque on a Current Loop in a Uniform Magnetic Field.

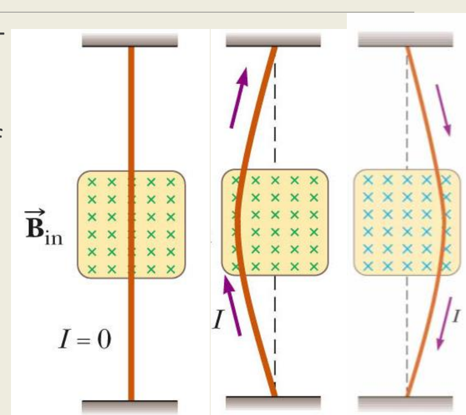
1.6 The Hall Effect.

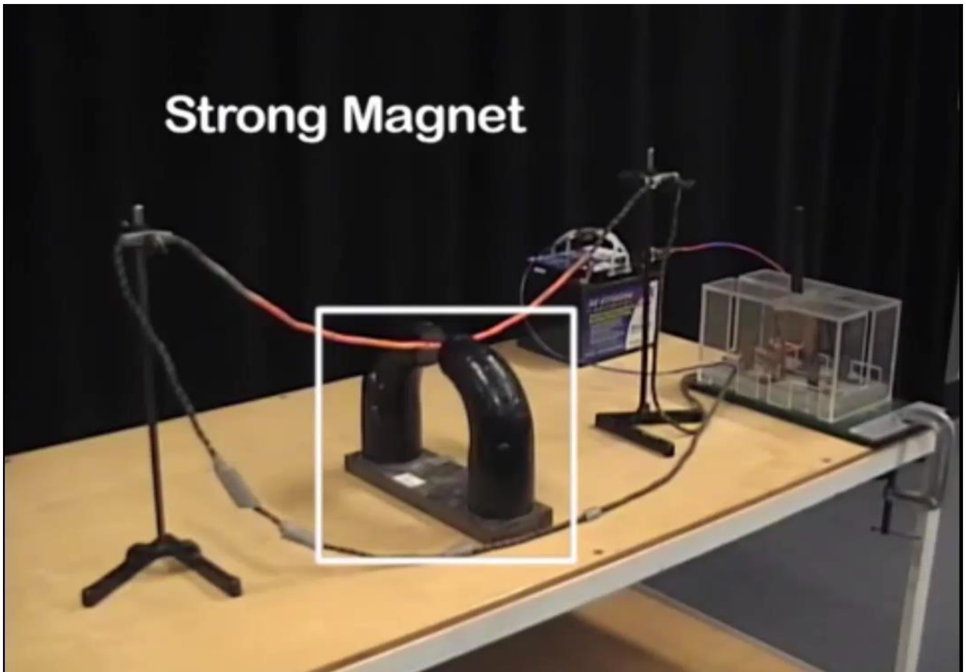
Magnetic Force on a Current Carrying Conductor, a wire

A force is exerted on a current-carrying wire placed in a magnetic field.

- The current is a collection of many charged particles in motion.

The direction of the force is given by the right-hand rule





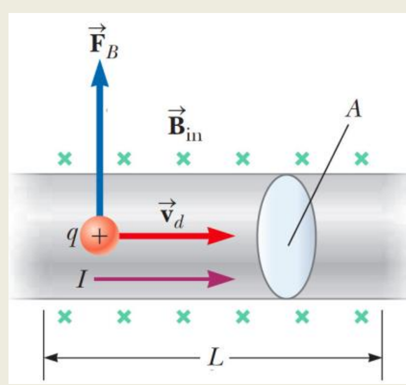
Force on a Wire, the equation

Consider a straight segment of wire of length L and cross-sectional area A carrying a current I in a uniform magnetic field **B.**

The magnetic force exerted on a charge q moving with a drift velocity $\mathbf{v}_d$.

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$$

To find the total force acting on the wire, we multiply the $q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$ force exerted on one charge by the number of charges in the segment.



Force on a Wire, the equation, continue

The number of charges in the segment is nAL, where n is the number of charges per unit volume. Hence, the total magnetic force on the segment of wire of length L is

$$\vec{\mathbf{F}} = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right) nAL$$

the current in the wire is $I = nqv_dA$. Therefore,

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

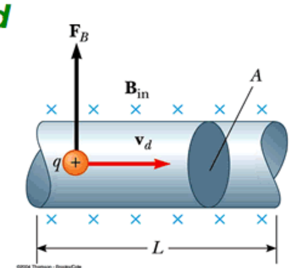
where **L** is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment. This expression applies only to a straight segment of wire in a uniform magnetic field.

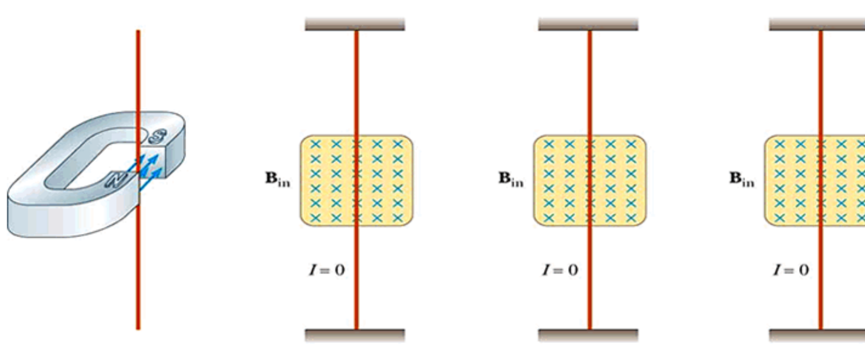
Magnetic Force **Magnetic Field**

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

current in wire

The moving electrons in the wire is immersed in an external B-Field and feels a magnetic force given by the right hand rule as shown.



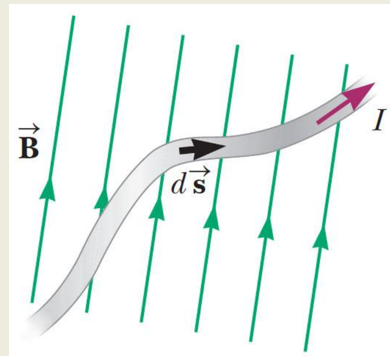


General Equation

Now consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field as shown in the Figure.

The magnetic force exerted on a small segment of vector length **ds** in the presence of a field **B** is,

$$d\vec{\mathbf{F}}_B = I d\vec{s} \times \vec{\mathbf{B}}$$

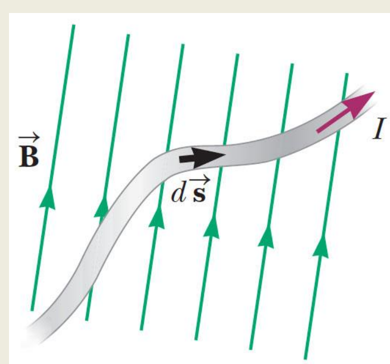


General Equation, continue

To calculate the total force $\mathbf{F_B}$ acting on the wire shown in the Figure, we integrate Equation over the length of the wire:

$$\vec{\mathbf{F}}_B = I \int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

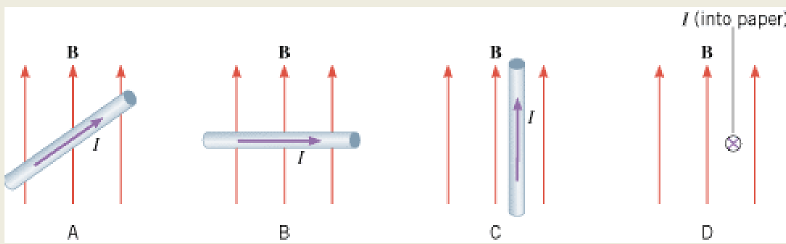
where a and b represent the endpoints of the wire.



Example 1

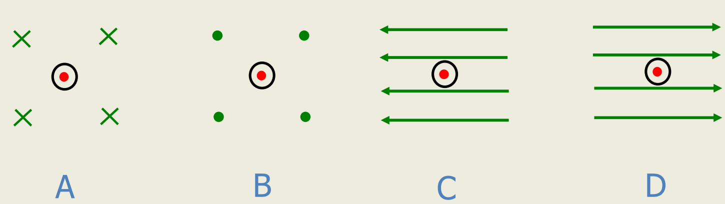
The same current-carrying wire is placed in the same magnetic field **B** in four different orientations.

Rank the orientations according to the magnitude of the magnetic force exerted on the wire, largest to smallest.



Example 2

A straight, horizontal length of copper wire is immersed in a uniform magnetic field. The current through the wire is out of page. Which magnetic field can possibly suspend this wire to balance the gravity?

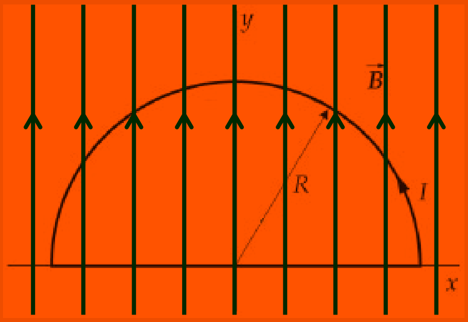


See problem 3 in problems to solve by your self..

Example 3

A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis as in the Figure.

Find (A) the magnitude and direction of the magnetic force acting on the straight portion of the wire and (B) on the curved portion.



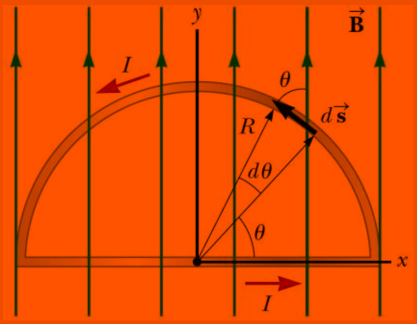
Solution

The force F_1 on the straight portion of the wire is out of the page.

$$\vec{\mathbf{F}}_1 = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

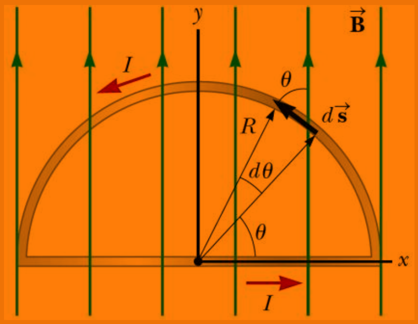
$$= I\int_{-R}^{R} B\,dx\,\hat{\mathbf{k}}$$

$$= 2IRB\,\hat{\mathbf{k}}$$



Solution, continue

The force F_2 on the curved portion is into the page.



$$d\vec{\mathbf{F}}_2 = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$= -IB \sin\theta \; ds \, \hat{\mathbf{k}}$$

$$ds = R \, d\theta$$

$$\vec{\mathbf{F}}_2 = -\int_0^{\pi} IRB \sin\theta \, d\theta \, \hat{\mathbf{k}}$$

$$= -IRB \int_0^{\pi} \sin\theta \, d\theta \, \hat{\mathbf{k}}$$

$$= -IRB[-\cos\theta]_0^{\pi} \, \hat{\mathbf{k}}$$

$$= IRB(\cos\pi - \cos 0)\hat{\mathbf{k}}$$

$$= IRB(-1 - 1)\hat{\mathbf{k}} \quad = \quad -2IRB\,\hat{\mathbf{k}}$$

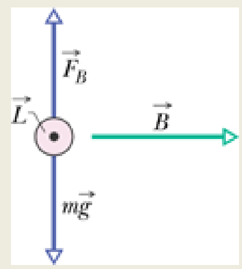
The force on the curved portion is the same in magnitude as the force on a straight wire between the same two points.

The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

Solve by your self

1. A conductor carrying a current I=15.0 A is directed along the positive x axis and perpendicular to a uniform magnetic field. A magnetic force per unit length of 0.120 N/m acts on the conductor in the negative y direction. Determine (a) the magnitude and (b) the direction of the magnetic field in the region through which the current passes.
2. A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, B = 1.60k T. If the current is in the positive x direction, what is the magnetic force on the section of wire?
3. A straight, horizontal length of copper wire has a current i=28 A through it. What are the magnitude and direction of the minimum magnetic field needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

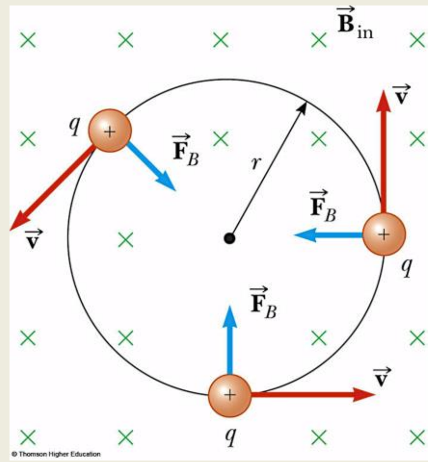


Charged Particle in a Magnetic Field

Consider a +ve charged particle moving in an external magnetic field with its velocity perpendicular to the field.

The magnetic force is always directed toward the center of the circular path.

The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle.



We use the particle under a net force model to write Newton's second law for the particle:

$$F = F_B = ma$$

Because the particle moves in a circle, we also model it as a particle in uniform circular motion and we replace the acceleration with centripetal acceleration:

$$F_B = qvB = \frac{mv^2}{r}$$

This expression leads to the following equation for the radius of the circular path:

$$r = \frac{mv}{qB}$$ **Radius of the circular path**

The **radius** of the path is proportional to the linear momentum *mv* of the particle and inversely proportional to the magnitude of the charge *q* on the particle and to the magnitude of the magnetic field B.

The angular speed of the particle

$$r = \frac{mv}{qB}$$

$$= \frac{v}{r} = \frac{qB}{m}$$ angular speed

The period of the motion (the time interval the particle requires to complete one revolution) is equal to the circumference of the circle divided by the speed of the particle:

$$T = \frac{2\ \ r}{v} = \frac{2}{\ } = \frac{2\ \ m}{qB}$$ period of the motion

These results show that the angular speed of the particle and the period of the circular motion do not depend on the speed of the particle or on the radius of the orbit.

> The angular speed ω is often referred to as the cyclotron frequency because charged particles circulate at this angular frequency in the type of accelerator called a cyclotron.

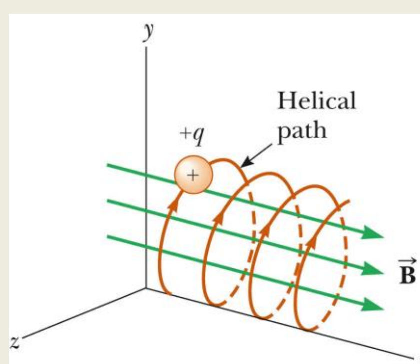
General Case

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to **B**, its path is a helix.

Same equations apply,

with

$$V = \sqrt{V_y^2 + V_z^2}$$



Example 1

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. **Find the speed of the proton.**

Solution

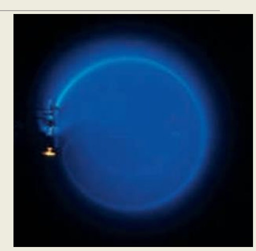
$$= \frac{v}{r} = \frac{qB}{m} \quad \Rightarrow \quad v = \frac{qBr}{m}$$

$$v = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(0.35\ \mathrm{T})(0.14\ \mathrm{m})}{1.67 \times 10^{-27}\ \mathrm{kg}}$$

$$= \boxed{4.7 \times 10^{6}\ \mathrm{m/s}}$$

Example 2

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm.



(A) What is the magnitude of the magnetic field?

(B) What is the angular speed of the electrons?

Solution (A) the magnitude of the magnetic field

$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}m_e v^2 - 0\right) + (q\,\Delta V) = 0$$

$$v = \sqrt{\frac{-2q\,\Delta V}{m_e}}$$

$$v = \sqrt{\frac{-2(-1.60 \times 10^{-19}\ \mathrm{C})(350\ \mathrm{V})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 1.11 \times 10^{7}\ \mathrm{m/s}$$

$$r = \frac{mv}{qB}$$

$$\boxed{B = \frac{m_e v}{er}}$$

$$B = \frac{m_e v}{er}$$

$$B = \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.11 \times 10^{7}\ \mathrm{m/s})}{(1.60 \times 10^{-19}\ \mathrm{C})(0.075\ \mathrm{m})} = 8.4 \times 10^{-4}\ \mathrm{T}$$

Solution B the angular speed of the electrons

$$\omega = \frac{v}{r} = \frac{1.11 \times 10^{7}\ \mathrm{m/s}}{0.075\ \mathrm{m}} = 1.5 \times 10^{8}\ \mathrm{rad/s}$$

$$\omega = (1.5 \times 10^{8}\ \mathrm{rad/s})(1\ \mathrm{rev}/2\pi\ \mathrm{rad}) = 2.4 \times 10^{7}\ \mathrm{rev/s}$$

Applications

- Velocity Selector
- Mass Spectrometer
- The Cyclotron

Applications involving charged particles moving in a magnetic field

In many applications, charged particles will move in the presence of both magnetic and electric fields.

In that case, the total force is the sum of the forces due to the individual fields.

In general (The Lorentz force):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Electric Force **Magnetic Force**

Velocity Selector

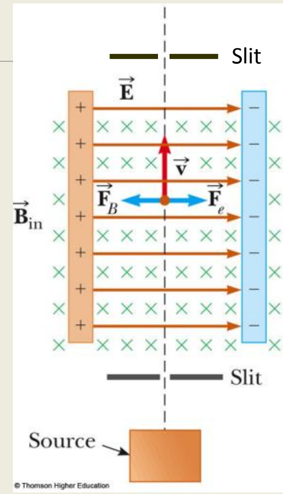
A uniform electric field is perpendicular to a uniform magnetic field.

When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line.

$$qE = qvB$$

This selects particles with velocities of the value

$$v = \frac{E}{B}$$



Mass Spectrometer

A mass spectrometer separates ions according to their mass-to-charge ratio.

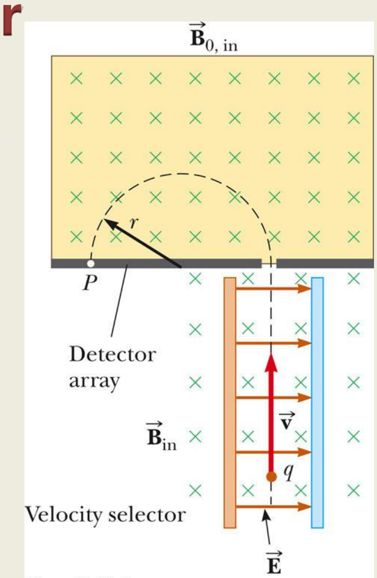
A beam of ions passes through a velocity selector and then enters a second magnetic field where the ions move in a semicircle of radius *r* before striking a detector at P.

From the equation

$$r = \frac{mv}{qB_o}$$

The ratio of m/q

$$\frac{m}{q} = \frac{rB_o}{v}$$



Mass Spectrometer

$$\frac{m}{q} = \frac{rB_o}{v}$$

The velocity is given by the velocity selector of the first part as

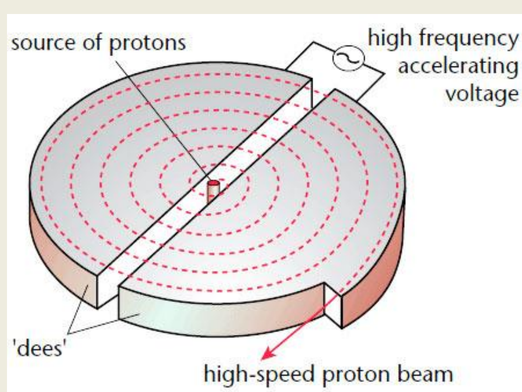
$$v = \frac{E}{B}$$

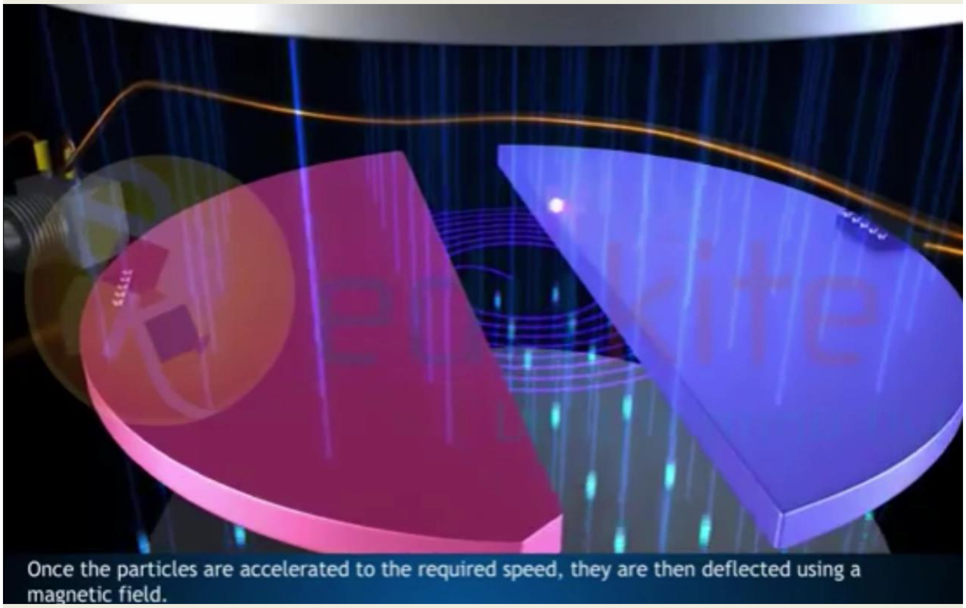
$$\frac{m}{q} = \frac{rB_o B}{E}$$

we can determine **m /q** by measuring the radius of curvature and knowing the field magnitudes **B, B$_0$** , and **E.**

The Cyclotron

A cyclotron is a device that can accelerate charged particles to very high speeds.





The Cyclotron

We can obtain an expression for the kinetic energy of the ion when it exits the cyclotron in terms of the radius R of the dees.

we know that

$$\frac{m}{q} = \frac{RB_o}{v} \longrightarrow v = \frac{qBR}{m}$$

the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

Solve by your self

1. An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2.00 mT. If the speed of the electron is 1.50×10^7 m/s, determine (a) the radius of the circular path and (b) the time interval required to complete one revolution.
2. An electron moves in a circular path perpendicular to a constant magnetic field of magnitude 1.00 mT. The angular momentum of the electron about the center of the circle is 4.00×10^{-25} kg.m^2/s. Determine (a) the radius of the circular path and (b) the speed of the electron.
3. Consider the mass spectrometer. The magnitude of the electric field between the plates of the velocity selector is 2.50×10^3 V/m, and the magnetic field in both the velocity selector and the deflection chamber has a magnitude of 0.035T. Calculate the radius of the path for a singly charged ion having a mass $m = 2.18 \times 10^{-26}$ kg.

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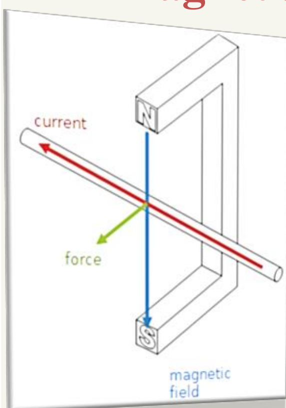
Magnetism and Alternating Current



Unit 1: Magnetic Fields

Lecture 3: Magnetic force acting a current-carrying conductor

Dr. Hazem Falah Sakeek
Al-Azhar University of Gaza

Unit 1: Magnetic Fields

1.1 Magnetic Fields and Forces.

1.2 Motion of a Charged Particle in a Uniform Magnetic Field.

1.3 Applications Involving Charged Particles Moving in a Magnetic Field.

1.4 Magnetic Force Acting on a Current-Carrying Conductor.

1.5 Torque on a Current Loop in a Uniform Magnetic Field.

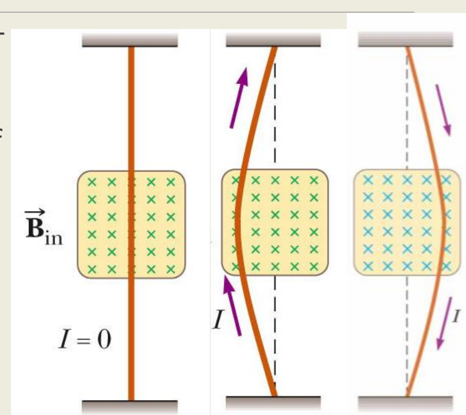
1.6 The Hall Effect.

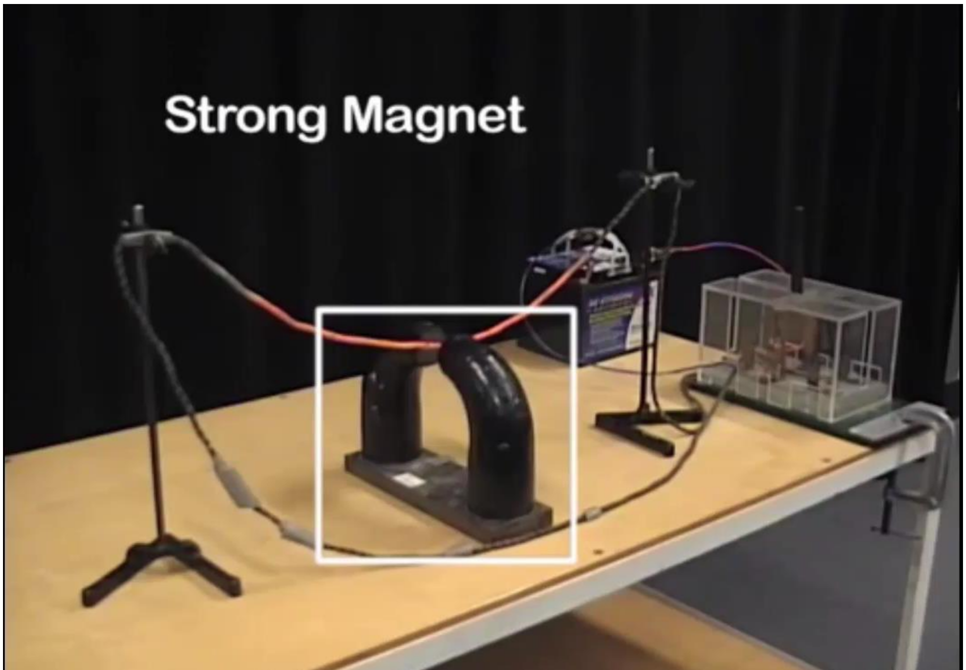
Magnetic Force on a Current Carrying Conductor, a wire

A force is exerted on a current-carrying wire placed in a magnetic field.

- The current is a collection of many charged particles in motion.

The direction of the force is given by the right-hand rule





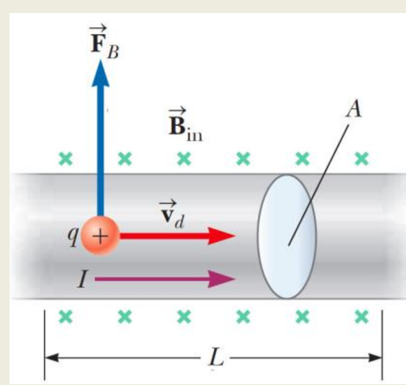
Force on a Wire, the equation

Consider a straight segment of wire of length L and cross-sectional area A carrying a current I in a uniform magnetic field **B.**

The magnetic force exerted on a charge q moving with a drift velocity $\mathbf{v}_d$.

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$$

To find the total force acting on the wire, we multiply the force $q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$ exerted on one charge by the number of charges in the segment.



Force on a Wire, the equation, continue

The number of charges in the segment is nAL, where n is the number of charges per unit volume. Hence, the total magnetic force on the segment of wire of length L is

$$\vec{\mathbf{F}} = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right) nAL$$

the current in the wire is $I = nqv_dA$. Therefore,

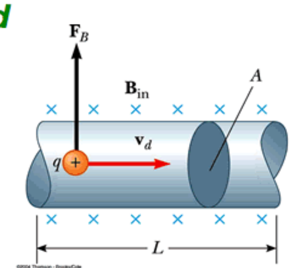
$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

where **L** is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment. This expression applies only to a straight segment of wire in a uniform magnetic field.

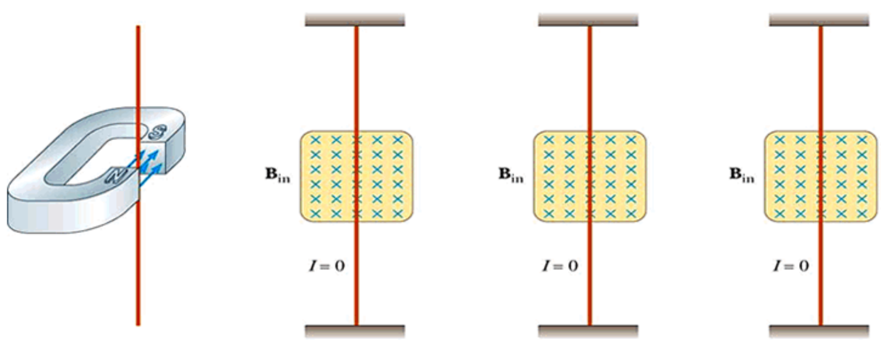
Magnetic Force **Magnetic Field**

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

current in wire



The moving electrons in the wire is immersed in an external B-Field and feels a magnetic force given by the right hand rule as shown.

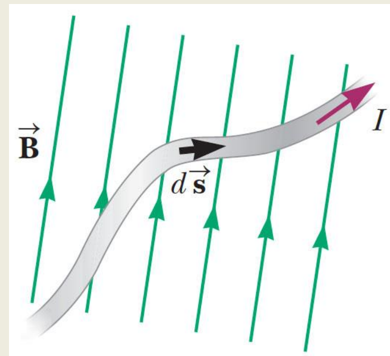


General Equation

Now consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field as shown in the Figure.

The magnetic force exerted on a small segment of vector length **ds** in the presence of a field **B** is,

$$d\vec{\mathbf{F}}_B = I d\vec{s} \times \vec{\mathbf{B}}$$

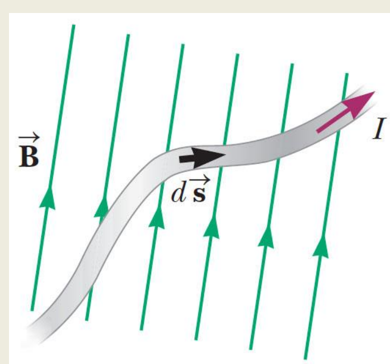


General Equation, continue

To calculate the total force $\mathbf{F_B}$ acting on the wire shown in the Figure, we integrate Equation over the length of the wire:

$$\vec{\mathbf{F}}_B = I \int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

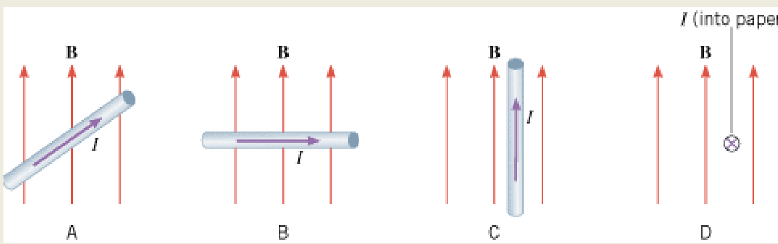
where a and b represent the endpoints of the wire.



Example 1

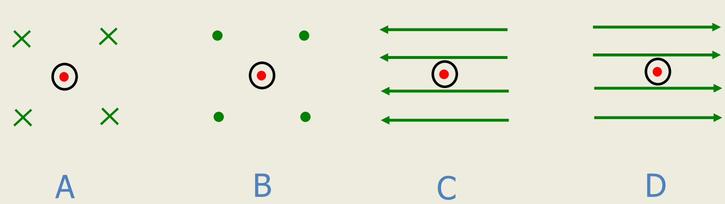
The same current-carrying wire is placed in the same magnetic field **B** in four different orientations.

Rank the orientations according to the magnitude of the magnetic force exerted on the wire, largest to smallest.



Example 2

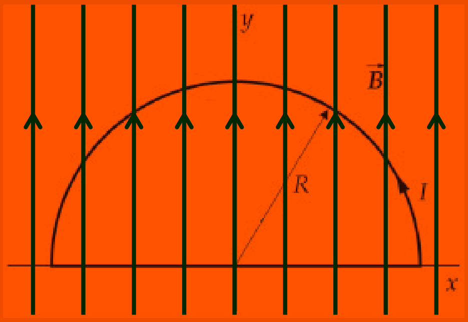
A straight, horizontal length of copper wire is immersed in a uniform magnetic field. The current through the wire is out of page. Which magnetic field can possibly suspend this wire to balance the gravity?



See problem 3 in problems to solve by your self..

Example 3

A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis as in the Figure.



Find (A) the magnitude and direction of the magnetic force acting on the straight portion of the wire and (B) on the curved portion.

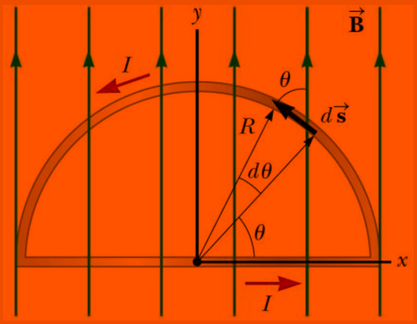
Solution

The force F_1 on the straight portion of the wire is out of the page.

$$\vec{\mathbf{F}}_1 = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

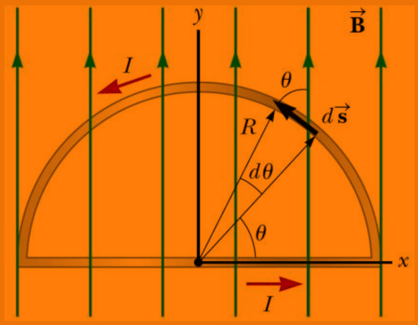
$$= I\int_{-R}^{R} B\,dx\,\hat{\mathbf{k}}$$

$$= \boxed{2IRB\,\hat{\mathbf{k}}}$$



Solution, continue

The force F_2 on the curved portion is into the page.



$$d\vec{\mathbf{F}}_2 = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$= -IB \sin\theta \, ds \, \hat{\mathbf{k}}$$

$$ds = R \, d\theta$$

$$\vec{\mathbf{F}}_2 = -\int_0^{\pi} IRB \sin\theta \, d\theta \, \hat{\mathbf{k}}$$

$$= -IRB \int_0^{\pi} \sin\theta \, d\theta \, \hat{\mathbf{k}}$$

$$= -IRB[-\cos\theta]_0^{\pi} \, \hat{\mathbf{k}}$$

$$= IRB(\cos\pi - \cos 0)\hat{\mathbf{k}}$$

$$= IRB(-1 - 1)\hat{\mathbf{k}} = -2IRB\,\hat{\mathbf{k}}$$

The force on the curved portion is the same in magnitude as the force on a straight wire between the same two points.

The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

Solve by your self

1. A conductor carrying a current I=15.0 A is directed along the positive x axis and perpendicular to a uniform magnetic field. A magnetic force per unit length of 0.120 N/m acts on the conductor in the negative y direction. Determine (a) the magnitude and (b) the direction of the magnetic field in the region through which the current passes.

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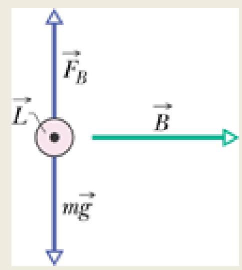
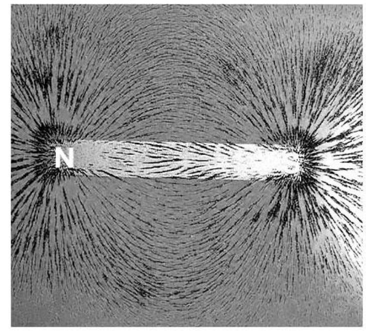


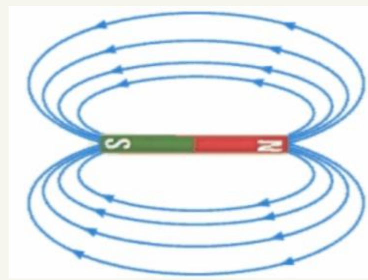
The magnetic field

A bar magnet has a magnetic field around it. This field is 3D in nature and often represented by lines LEAVING north and ENTERING south

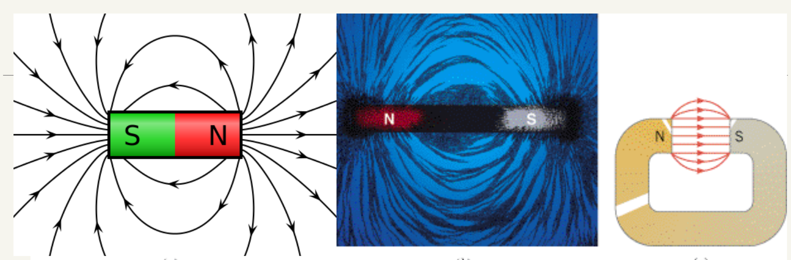
The magnetic field is a vector that has both magnitude and direction.

The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.





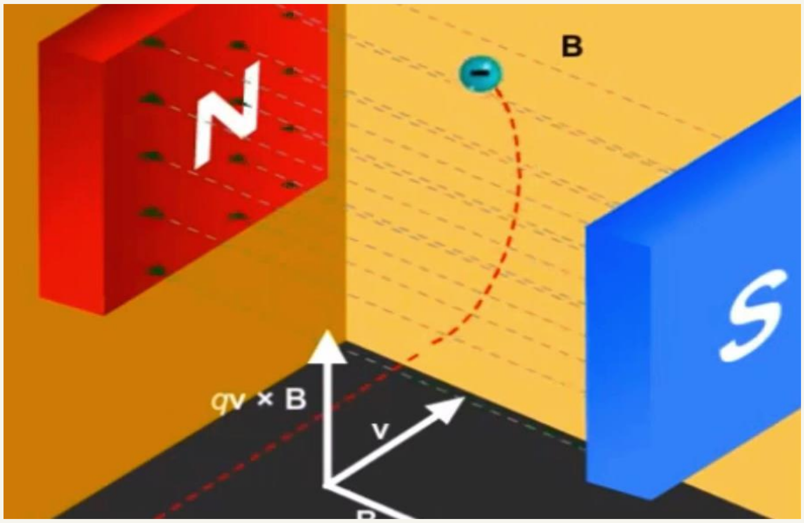
The properties of magnetic field line



(a) (b) (c)

1. The lines originate from the north pole and end on the south pole; they do not start or stop in mid-space.
2. The magnetic field at any point is tangent to the magnetic field line at that point.
3. The strength of the field is proportional to the number of lines per unit area that passes through a surface oriented perpendicular to the lines.
4. The magnetic field lines will never come to cross each other.

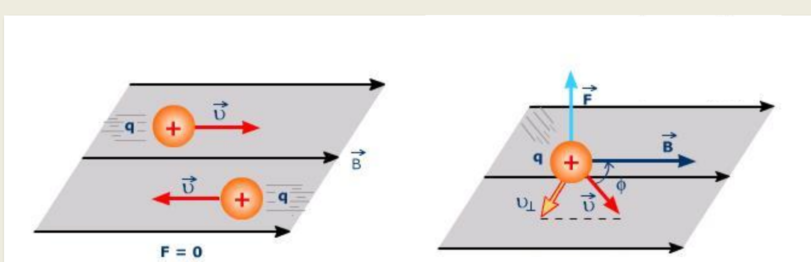
Magnetic force on moving charge



Magnetic force on moving charge

When a charge is placed in a magnetic field, it experiences a magnetic force if two conditions are met:

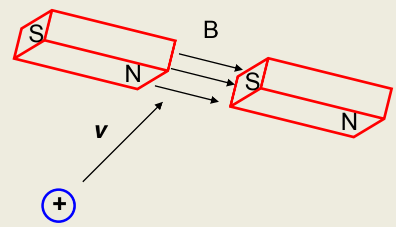
1. The charge must be moving. No magnetic force acts on a stationary charge.
2. The velocity of the moving charge must have a component that is perpendicular to the direction of the field.



Properties of the magnetic force on a charged particle moving in a magnetic field

We can define a magnetic field **B** at some point in space in terms of the magnetic force F_B the field exerts on a charged particle moving with a velocity **v**, which we call the test object.

Experiments on various charged particles moving in a magnetic field give the following results:

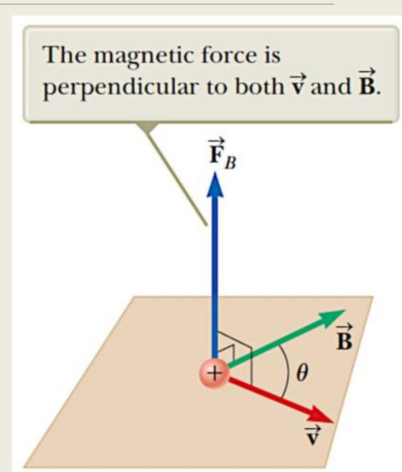


(1) The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.

Properties of the magnetic force on a charged particle moving in a magnetic field

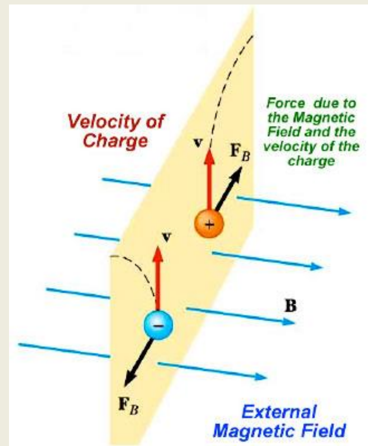
(2) When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.

(3) When the particle's velocity vector makes any angle θ ≠ 0 with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and ***B***; that is, F_B is perpendicular to the plane formed by **v** and **B.**



Properties of the magnetic force on a charged particle moving in a magnetic field

(4) The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.



Properties of the magnetic force on a charged particle moving in a magnetic field

(5) The magnitude of the magnetic force exerted on the moving particle is proportional to sinθ, where θ is the angle the particle's velocity vector makes with the direction of **B**.

$$F_B = qvB\sin$$

Vector expression for the magnetic force on a charged particle moving in a magnetic field

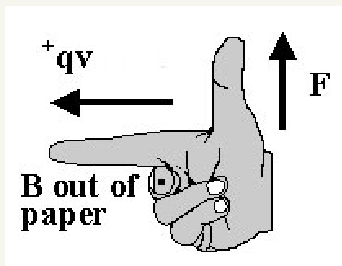
Magnetic Force   Magnetic Field

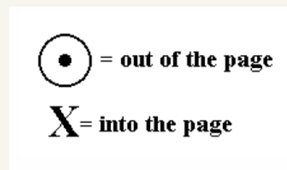
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

velocity of charge

Direction of the magnetic force?
Right Hand Rule

To determine the DIRECTION of the force on a **POSITIVE** charge we use a special technique that helps us understand the 3D perpendicular nature of magnetic fields.





Unit of Magnetic Field

SI unit of magnetic field is the newton per coulomb-meter per second, which is called the tesla (T):

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C.m/s}}$$

Because a coulomb per second is defined to be an ampere,

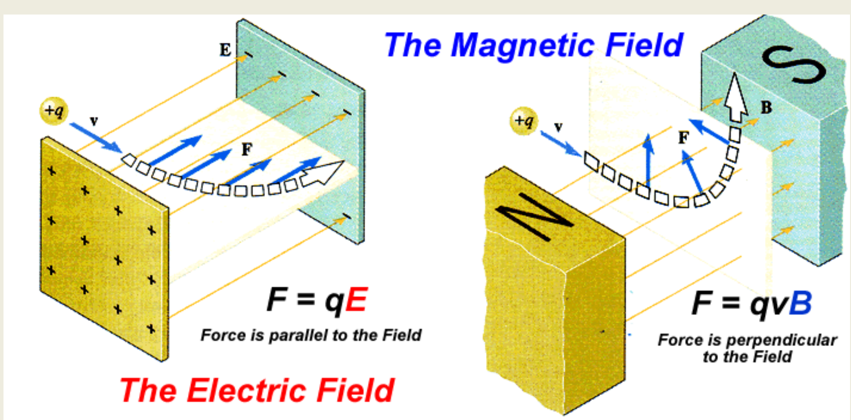
$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A.m}}$$

A non-SI magnetic-field unit in common use, called the gauss (G), is related to the tesla through the conversion 1 T = 10^4 G.

Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	**30**
Strong conventional laboratory magnet	**2**
Medical MRI unit	**1.5**
Magnetic Bar	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain due to nerve impulses	10^{-13}

Motion of charge particle in
- Electric field
- Magnetic field



Differences between Electric and Magnetic Forces

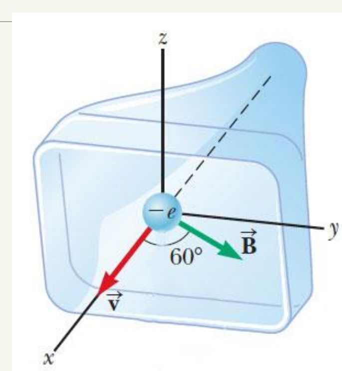
1. The electric force vector is along the direction of the electric field, whereas the magnetic force vector is perpendicular to the magnetic field.
2. The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
3. The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application.

The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. The field can alter the direction of the velocity vector, but it cannot change the speed or kinetic energy of the particle.

Example 1

An electron in an old-style television picture tube moves toward the front of the tube with a speed of 8.0×10^{6} m/s along the x axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the x axis and lying in the xy plane.

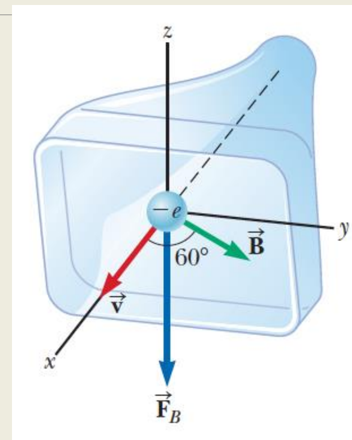
Calculate the magnetic force on the electron.



Solution

Use one of the right-hand rules to determine the direction of the force on the electron

$$F_B = qvB\sin$$

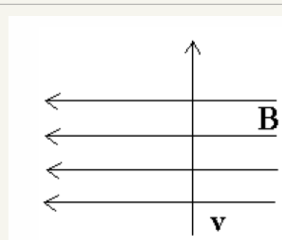


= **(1.6 × 10^{-19} C)(8.0 × 10^{6} m/s)(0.025 T)(sin 60^{o})**

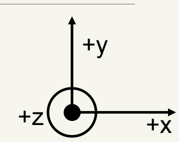
= 2.8 × 10^{-14} N

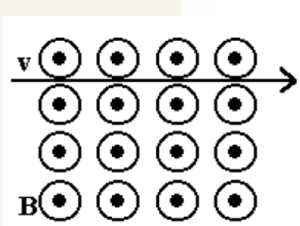
Example 2

Determine the direction of the unknown variable for a proton moving in the field using the coordinate axis given

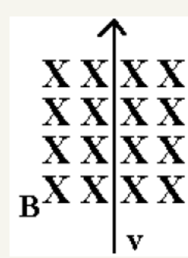


B = -x
v = +y
F = **+z**





B =+Z
v = +x
F = **-y**



B = -z
v = +y
F = **-x**

Example 3

A spatially uniform magnetic field cannot exert a magnetic force on a particle in which of the following circumstances? There may be more than one correct statement.

- (a) The particle is charged.
- (b) The particle moves perpendicular to the magnetic field.
- (c) The particle moves parallel to the magnetic field.
- (d) The magnitude of the magnetic field changes with time.
- (e) The particle is at rest.

Example 4

A particle with electric charge is fired into a region of space where the electric field is zero. It moves in a straight line. Can you conclude that the magnetic field in that region is zero?

- (a) Yes, you can.
- (b) No; the field might be perpendicular to the particle's velocity.
- (c) No; the field might be parallel to the particle's velocity.
- (d) No; the particle might need to have charge of the opposite sign
- to have a force exerted on it.
- (e) No; an observation of an object with electric charge gives no information about a magnetic field.

Example 5

Classify each of the following statements as a characteristic (a) of electric forces only, (b) of magnetic forces only, (c) of both electric and magnetic forces, or (d) of neither electric nor magnetic forces.

(1) The force is proportional to the magnitude of the field exerting it.

(2) The force is proportional to the magnitude of the charge of the object on which the force is exerted.

(3) The force exerted on a negatively charged object is opposite in direction to the force on a positive charge.

(4) The force exerted on a stationary charged object is nonzero.

(5) The force exerted on a moving charged object is zero.

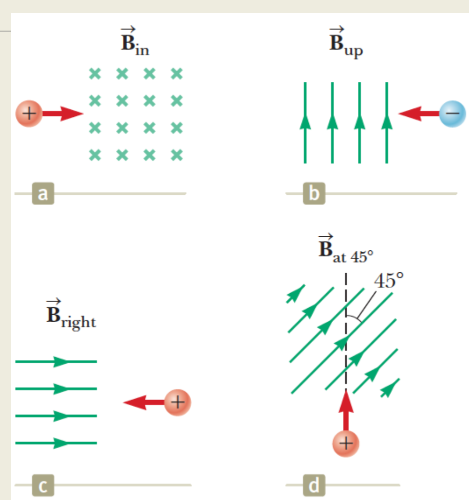
(6) The force exerted on a charged object is proportional to its speed.

(7) The force exerted on a charged object cannot alter the object's speed.

(8) The magnitude of the force depends on the charged object's direction of motion.

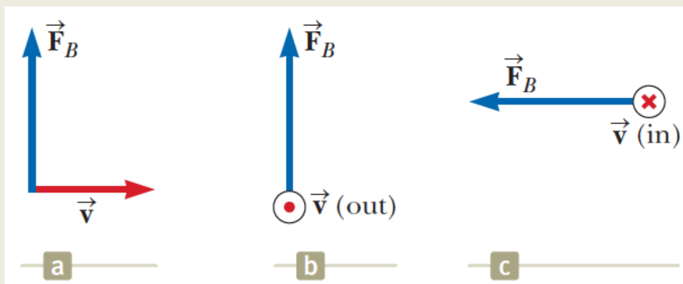
Example 6

Determine the initial direction of the deflection of charged particles as they enter the magnetic fields shown in the Figure



Example 7

Find the direction of the magnetic field acting on a positively charged particle moving in the various situations shown in the Figure if the direction of the magnetic force acting on it is as indicated.



Solve by Your self

- Two charged particles are projected in the same direction into a magnetic field perpendicular to their velocities. If the particles are deflected in opposite directions, what can you say about them?
- How can the motion of a moving charged particle be used to distinguish between a magnetic field and an electric field?
- Can a constant magnetic field set into motion an electron initially at rest? Explain your answer.